

Adventures in Electricity Finance

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Open PDE problem

Solve, on $t > 0$, $\Theta(t) < S < \infty$

$$V_t + \frac{1}{2}\sigma^2 S^2 V_{SS} + rSV_S - rV$$

With final condition

$$V(T, S) = \max(K - S, 0)$$

And boundary conditions

$$V(\infty, t) = 0,$$

$$V(\Theta(t), t) = K - \Theta(t); \quad V_S(\Theta(t), t) = -1$$

Nonlinear problem with no closed form solution.

Exercise boundary (David Itkin)

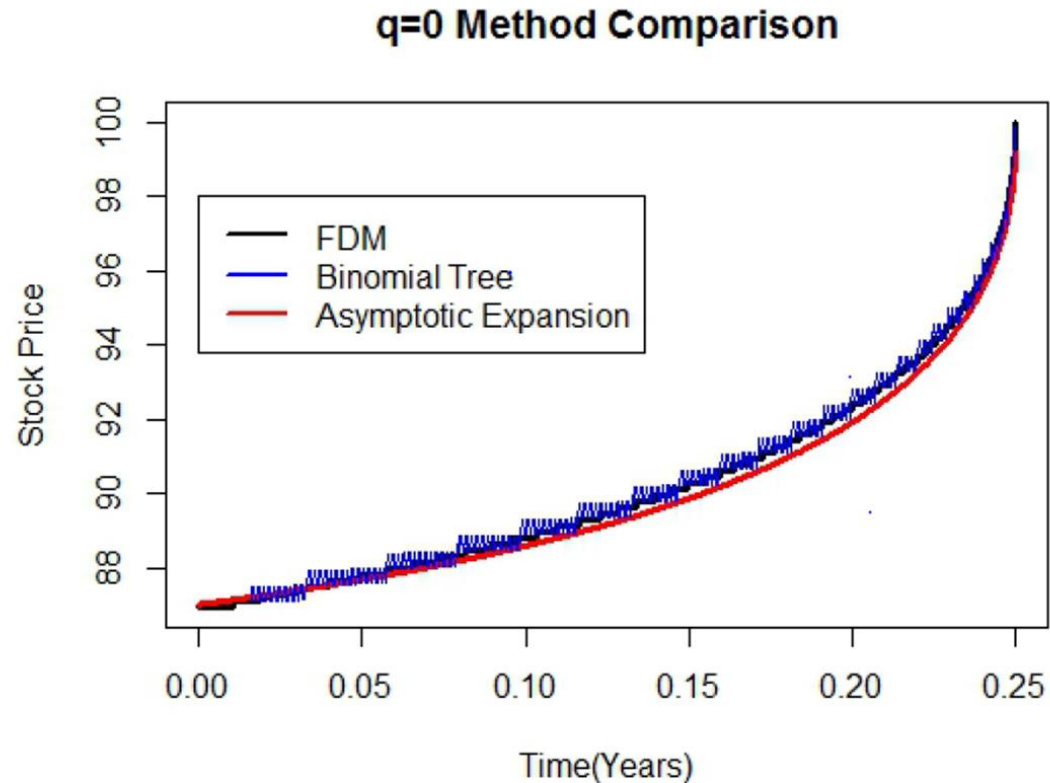
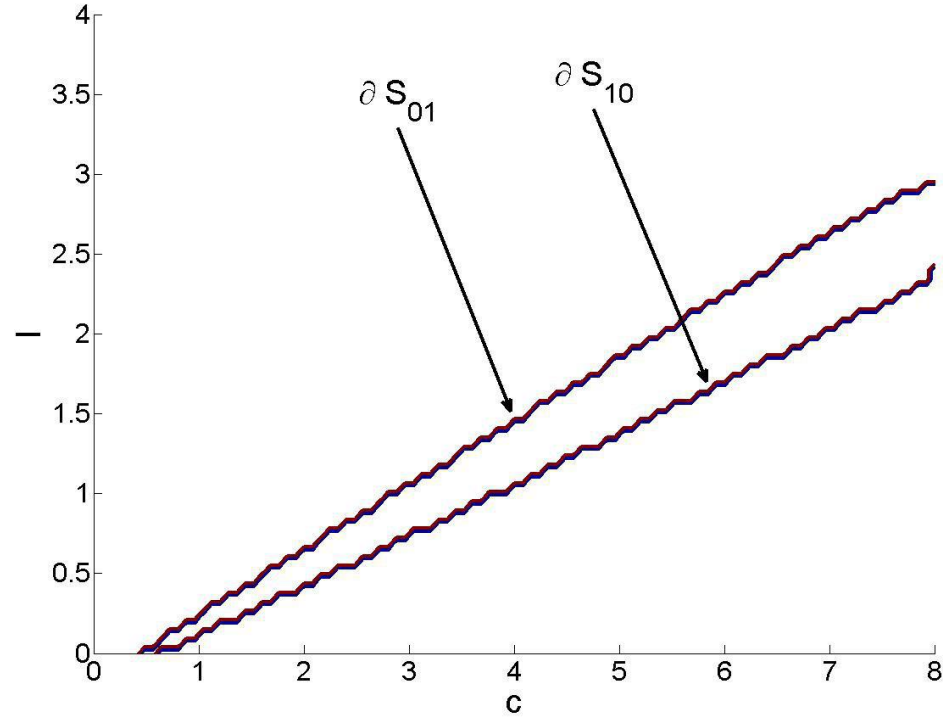


Figure 1: $r = 0.05$, $\sigma = 0.2$, $T = 0.25$, $K = 100$

Corn Ethanol S_{01} turn on, S_{10} turn off (Chris Maxwell)



Some goals

- What can financial math tell us about incentives?
- Give some insight about what is happening behind the headlines.
- I also want to “be discrete” and have some fun talking about nonlinear difference equations.
- And give some intuition about the PDE we opened with.

Collaborators

- Lindsay Anderson: Cornell University
- Natasha Burke (Kirby): Royal Bank Energy Trading – former AM PhD student
- Jill (Zhongwen) Hu, B.Sc. summer student, Western University Canada
- Mark Somppi, Goldcorp
- Yu Sun, M.Sc. – research assistant
- All data from IESO (economic/load), Environment Canada (weather)

The societal problem

- Renewables (Wind, Solar, small Hydro) are the cornerstone of green power initiatives both in Ontario and worldwide.
- Wind “penetration” has increased dramatically in recent years
- But wind and other renewables require expensive subsidies and do not result in dispatchable power.

Two linked problems

- How to manage this new system?
- How to pay for it?

Wind: increasing in Ontario (www.windontario.ca)

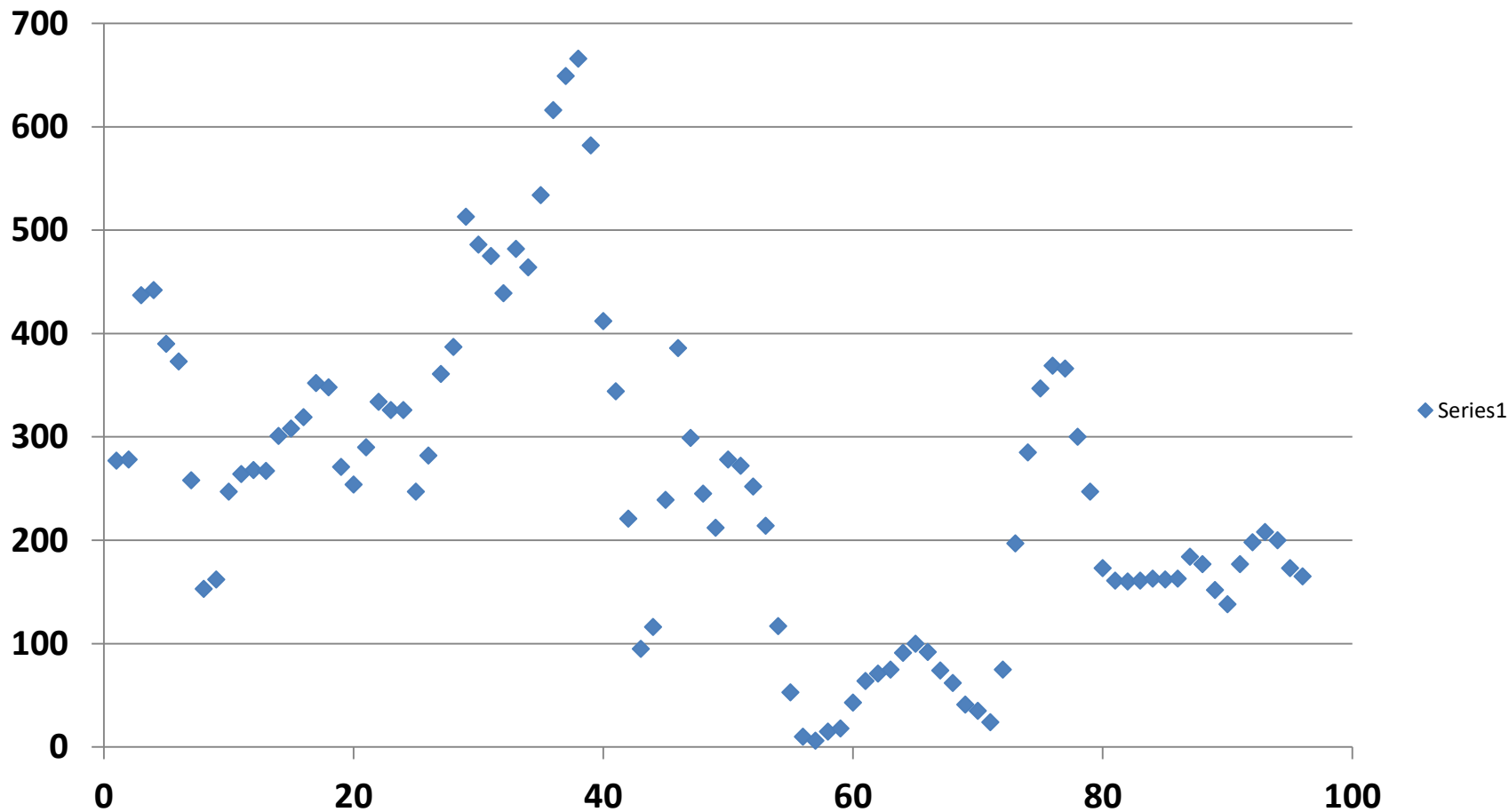


Installed generation (Ontario) (IESO, Dec 2017)

Resource Type	Capacity	Annual Output
Nuclear	13,009 MW (35%)	91.7 TWh (61%)
Gas/Oil	10,277 MW (28%)	12.7 TWh (9%)
Hydro	8,480 MW (23%)	35.7 TWh (24%)
Wind	4,213 MW (11%)	9.3 TWh (6%)
Biofuel	495 MW (1%)	0.49 TWh (< 1%)
Solar	380 MW (1%)	0.46 TWh (< 1%)

Hourly Ontario wind production (MW):

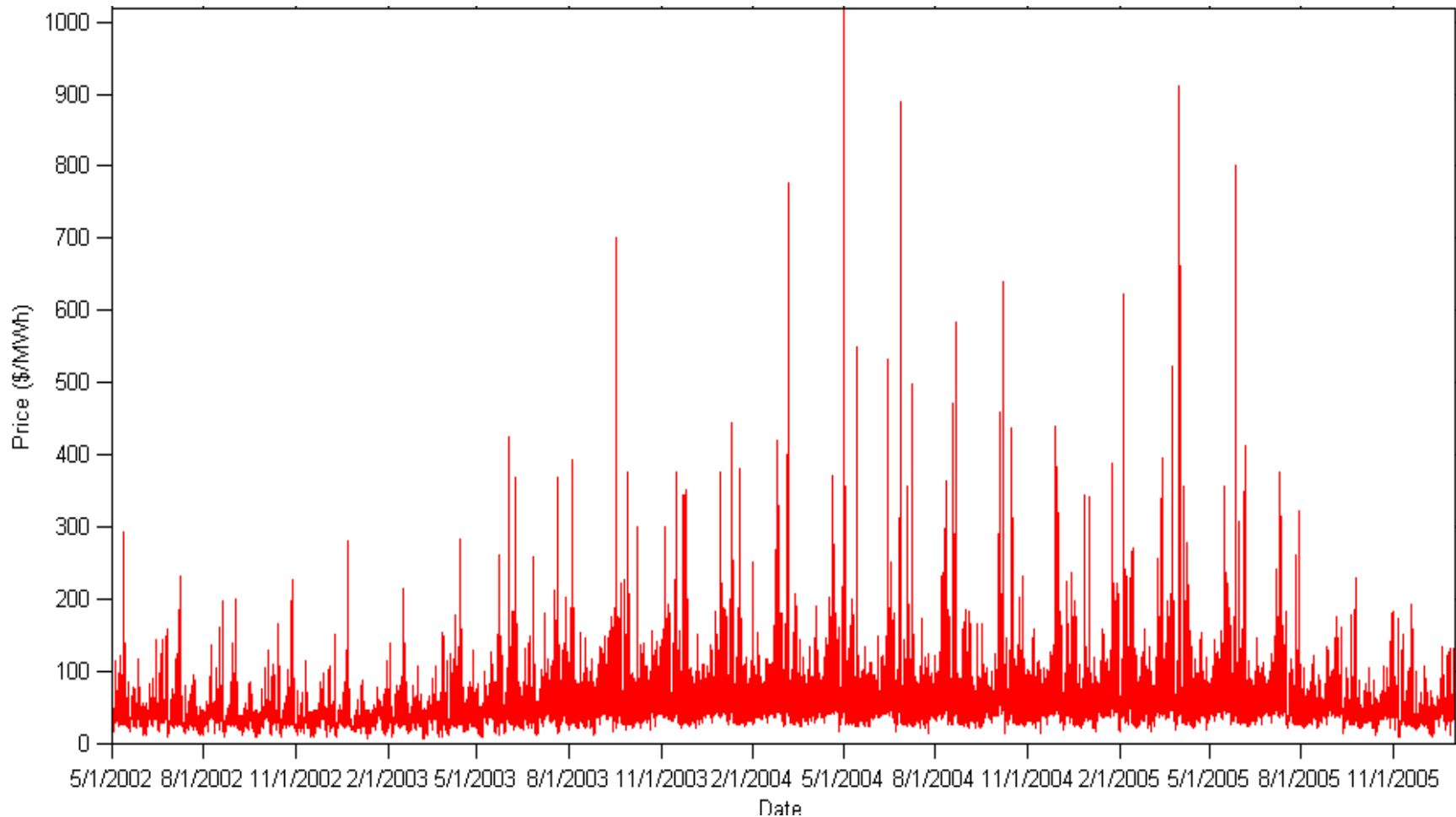
May 5 – May 8 2011. Source: IESO



Market impact

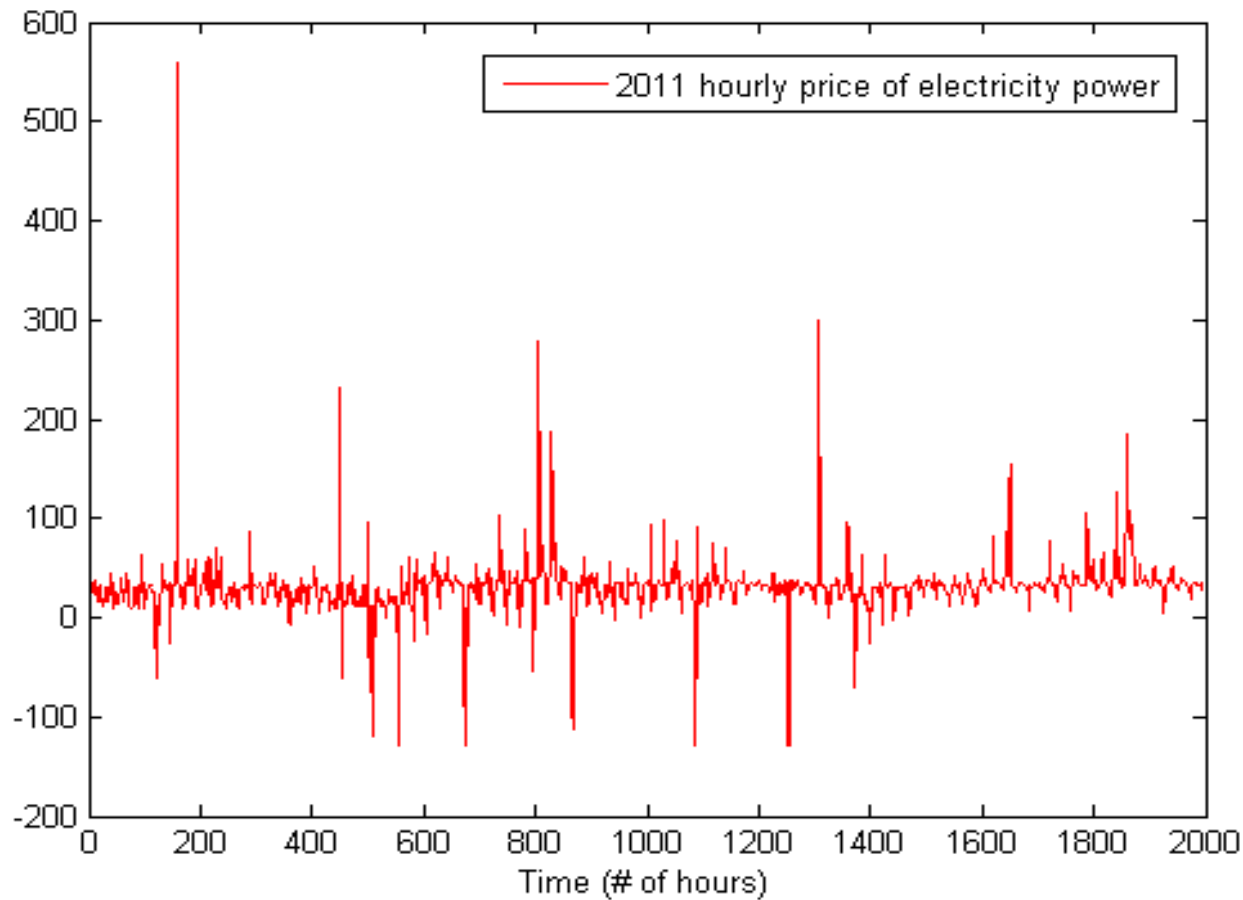
- Greater price instability
- Frequent negative prices

Ontario Open Market Price: Old days (next to no renewables)



Ontario Electricity Price, \$/MWh

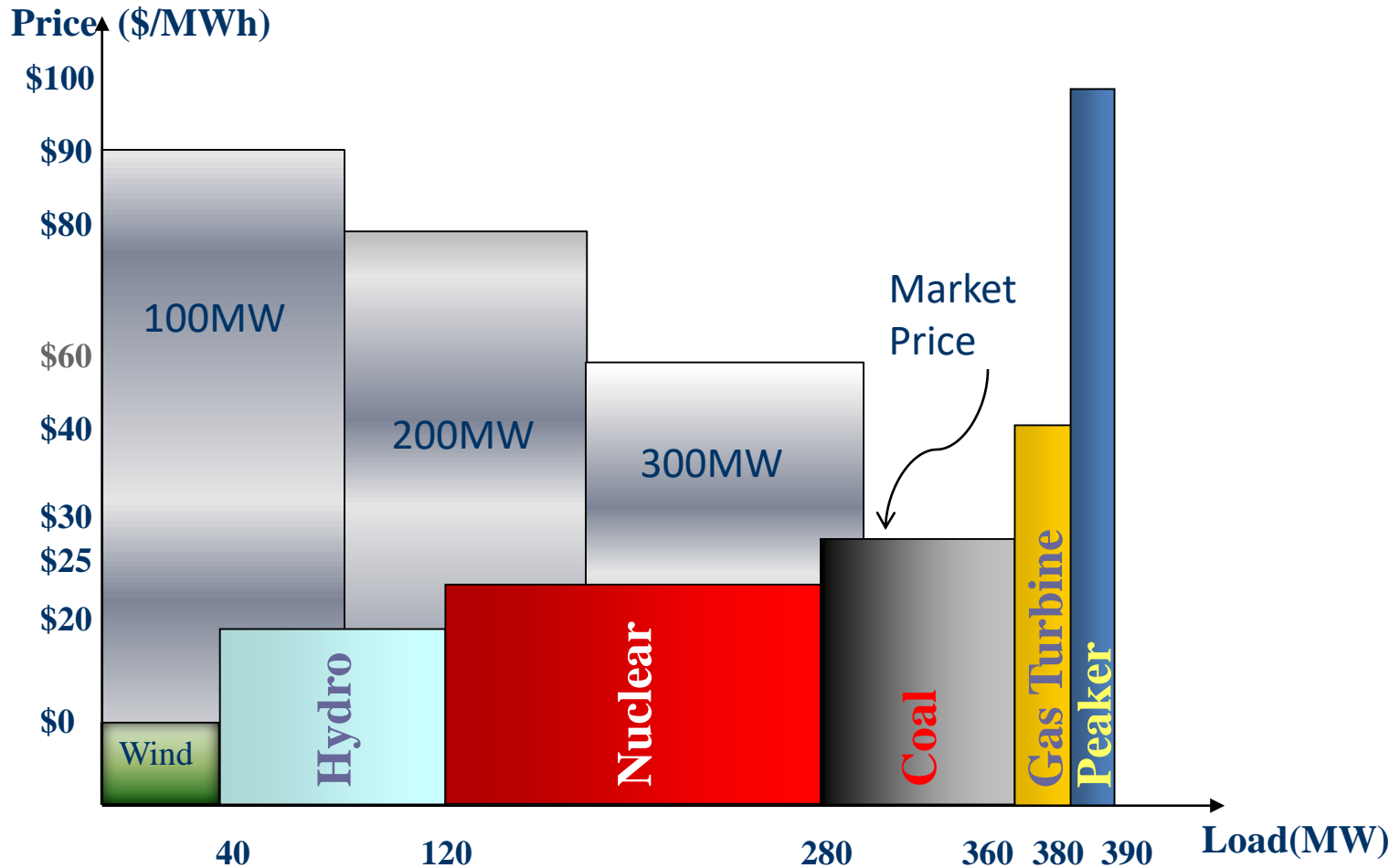
Source IESO,



How does the Ontario market work?

- Power is traded at each of 24 hours per day.
- Generators offer power; users bid for power.
- Bids/Offerers are prepared by 11PM the previous night for each hour but can be revised up until 4 hours ahead of the beginning of each hour.
- Each participant submits one or more ordered pairs into the market for that hour – (amount bid, price bid) or (amount offered, price offered).

Stack-Based Pricing



Bid/Ask strategy

- If you “have to have it” you bid a very high amount.
- If you “have to sell it” you bid a very low, often a negative, amount. For instance Nuclear Power Plant.
- Wind/Solar guaranteed sell, and also bid low.
- Price is set by the flexible people.

Is the solution storage?

- Is energy storage the solution?
- Storage could buffer uncertainties.
- But storage is expensive!
- In current Ontario market, impossible to bill.
- Look at market rules to incent storage

Impact of regulation on storage

- Economic incentives to avoid excessive offering of power illustrated by simple model of a storage facility with just 4 parameters:
 - the value of a unit of power (M),
 - the fractional loss in storing the power (γ)
 - the probability of generating the power (p),
 - and the penalty from bidding power into the market that isn't delivered (x).

The model

- The model generates a nonlinear system of difference equations that can be solved in closed form allowing many insights.
- Details in “Optimal Management of Wind Energy with Storage: Structural Implications for Policy and Market Design”, Lindsay Anderson, Natasha Burke, Matt Davison, *Journal of Energy Engineering* (2014)

Our model: Wind meteorology

- A wind producer produces $\$M$ of electricity if it is windy, otherwise nothing.
- Each period it is windy with probability p and calm with probability $1-p$, $0 \leq p \leq 1$.
- No access to informative forecasts

Market Rules

- The wind producer must decide in advance whether to offer power into the market.
- If power is offered and it is windy, producer gets \$M.
- If power is not offered it cannot be sold, whether or not it is windy.

Penalties

- If the producer offers power and can't deliver she must pay a penalty of $-xM$; $x \geq 0$
- Ontario market: $x = 0$;
- New York market: $x > 0$.

The storage

- The wind producer has access to a storage facility sized to store a single unit of wind energy. This storage can be filled or withdrawn in a single hour.
- Storage is “lossy” and we assess the cost of this loss at withdrawal. If a unit of energy is withdrawn it earns $(1-\gamma)M$, $0 \leq \gamma \leq 1$.

Storage contracts

- Storage facilities are leased for N periods.
- At the end of the lease, storage is returned to its owner.
- If full, facility gets a cash refund of $(1-\gamma)M$,
- Return empty facility: get nothing.

$V(F,k)$ and $V(E,k)$

- The value of a full storage facility, assuming optimal operation, with k periods left before the end of the lease is denoted by $V(F,k)$
- The value of an empty storage facility, assuming optimal operation, with k periods left before the end of the lease is $V(E,k)$

$V(F,B,k)$ and $V(F,N,k)$

- With k periods remaining we must decide whether to offer power or not.
- The value of a full facility with k periods remaining given we offer is $V(F,B,k)$
- If we don't offer power the value of the full facility with k periods remaining is $V(F,N,k)$.
- $V(F,k) = \max[V(F,B,k), V(F,N,k)]$

$V(E,B,k)$ and $V(E,N,k)$

- The value of an empty facility with k periods remaining given that we offer is $V(E,B,k)$
- If we don't offer power the value of the full facility with k periods remaining is $V(E,N,k)$.
- $V(E,k) = \max[V(E,B,k), V(E,N,k)]$

The recursion relation: empty

- We use dynamic programming to solve this.
- We've already 'turned around' time by describing everything in terms of time remaining.
- $V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$
- $V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$
- Since if you don't bid and it's empty, you might as well fill the facility to sell later.

The recursion relation: full

- $V(F,N,k) = pV(F,k-1) + (1-p)V(F,k-1) = V(F,k-1)$.
- $V(F,B,k)$: if we bid and it's not windy we can choose whether to pay the penalty or empty the storage. Hence:
 - $V(F,B,k) = p[M + V(F,k-1)]$
 $+ (1-p)\max[-xM + V(F,k-1), (1-\gamma)M + V(E,k-1)]$

A note on expectations

- It probably makes sense to optimize the expected value of the cash flows as done above since the procedure will be repeated many times.
- If you want to add risk aversion, that will only have the effect of distorting the probability, so replace p by q and the structure of the equations remains.

System 0

$$V(F,B,k) = p[M+V(F,k-1)] \\ + (1-p)\max[-xM+V(F,k-1), (1-\gamma)M+V(E,k-1)]$$

$$V(F,N,k) = V(F,k-1)$$

$$V(F,k) = \max[V(F,B,k), V(F,N,k)]$$

$$V(E,B,k) = p[M+V(E,k-1)] + (1-p)[-xM+V(E,k-1)]$$

$$V(E,N,k) = pV(F,k-1) + (1-p)V(E,k-1)$$

$$V(E,k) = \max[V(E,B,k), V(E,N,k)]$$

$$V(F,0) = (1-\gamma)M, \quad V(E,0) = 0.$$

Solving this

- Solution of this system requires at each time:
- **Optimal bidding rules**, when empty and when full, before we know if the wind will blow or not.
- The **optimal decision** about whether to pay the **penalty or empty** the storage in the full, bid, no wind case.
- Expressions for **$V(F,k)$ and $V(E,k)$** .

Solving for $V(F,k)-V(E,k)$

System is really just one dimensional:

$$\text{Set: } V(F,k)-V(E,k) = (1-\gamma)M + Mm(k)$$

Then

$$m(k) = m(k-1) + \min[x(1-p), \gamma p - pm(k-1)] \\ - \min[p, x(1-p), (1-p)m(k-1)]$$

It's easy to check that, $m(k) = \min[\gamma p, x(1-p^k)]$

$$V(F,k)-V(E,k) = (1-\gamma)M + \min[p\gamma M, (1-p^k)xM]$$

Optimal control

- If $x \geq \gamma^* \max[1, p/(1-p)]$ then the optimal control is to bid when full and not bid when empty.
- $x \leq p\gamma$: small penalties you always bid whether you are full or empty
- Sufficiently huge penalties are never collected, and small penalties don't change behaviour, although may raise "tax like" revenue.

Empty bid rules: medium penalties

- $k^* < \ln[1-\gamma p/x]/\ln(p)$ –bid when empty, 0
- So you don't bid (sufficiently far from maturity)
and then bid (sufficiently close to maturity)
- Can also compute facility values.

Facility values

- When $x \geq \gamma^* \max[1, p/(1-p)]$
- $V(E, k) = kp[1-\gamma(1-p)]M$ ($k \geq 0$).
- $V(F, k) = (kp+1)[1-\gamma(1-p)]M$ ($k \geq 1$).

MEDIUM – depends on k^* again.

- When $x \leq p\gamma$
- Or $V(E, k) = k[p-x(1-p)]M$
- $V(F, k) = k[p-x(1-p)]M + (1-\gamma)M + x(1-p^k)M$.

To find value of storage

- Need to compare $V(E,k)$ with $W(k)$, is the value of this turbine (no storage) with k periods remaining.
- $W(k)$ follows the difference equation:
- $W(k) = \max[W(B,k), W(N,k)]$
- $W(B,k) = p[M + W(k-1)] + (1-p)[-xM + W(k-1)]$
- $W(N,k) = pW(k-1) + (1-p)W(k-1) = W(k-1)$
- So $W(k) = W(k-1) + \max[p-x(1-p), 0] * M$; $W(0) = 0$.
- So **$W(k) = kM * \max[p-x(1-p), 0]$** .

Conclusions

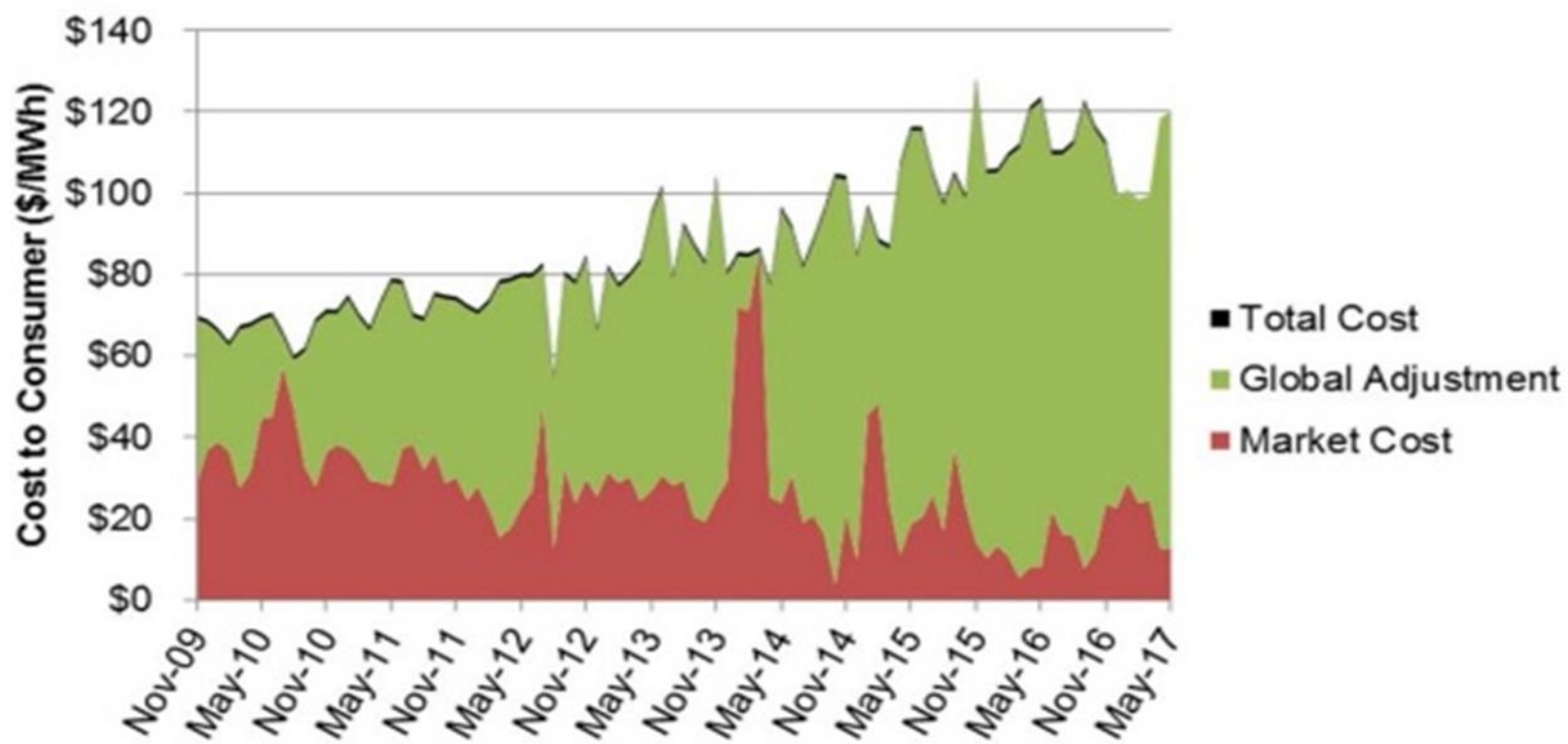
- A simple model can be exactly solved and shows some interesting intuition.
- Model has many operational and economic policy insights, but for today point is that the valuation/operation problem yields a nonlinear equation.
- Weather forecasts destroy value of storage.
- Interesting “Value of information” literature
- See also Zhao & Davison, *J. Hydrology*, in an energy finance setting (hydrological inflow forecasts, hydro dam operation).

Problem 2: How do we pay for it?

- Current NSERC Engage project with Goldcorp

Recall Renewables got high prices

- Solar power generators are guaranteed \$443/MWh for all power they sell; wind and special green microhydro \$140/MWh.
- But they are* must-dispatch and always bid.
- Meshing these guarantees with the market rules has “broken” the market.
- Cost of this must be absorbed somehow.
- That is called global adjustment



The Goldcorp game

- That's an attempt to shift the cost of global adjustment from the consumers to big producers.
- An outcome might be to flatten load shape, but this is not likely.
- Goldcorp can shut the mine/mill, but is also investigating energy storage. (better than diesel which is expensive in a fly in setting).

Bizzarre Market Rule

- Large Industrial Power Users (> 5MW) are billed, at the end of the year, for how much power they used on the top 5 power demand hours (for the entire province); each demand hour coming on a different day.
- Cost is substantial.

A strange game

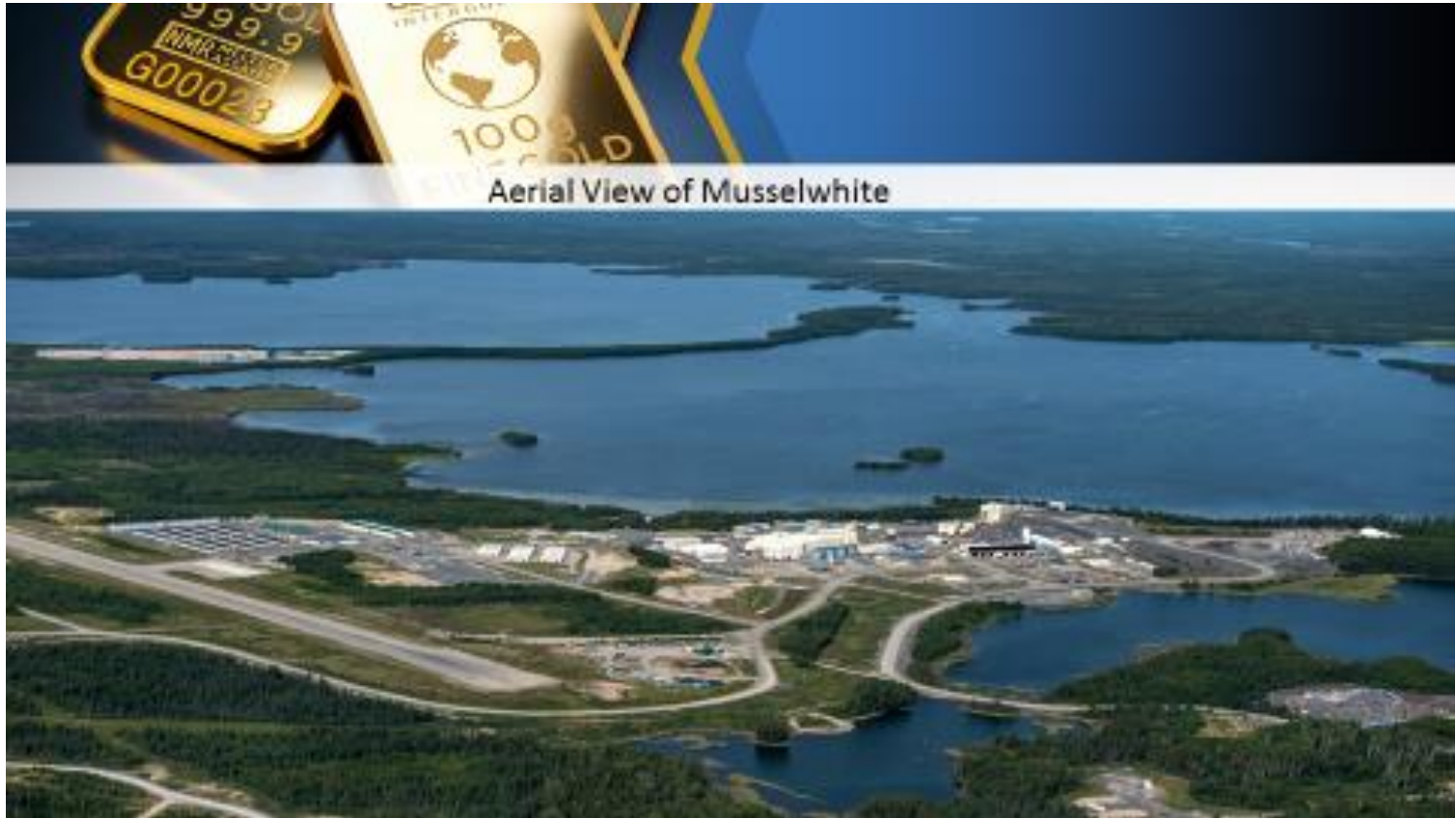
- Draw N random variables in succession:
 $x_N, x_{N-1}, \dots, x_2, x_1$. (numbered in reverse order).
- After each variable is drawn the player can decide whether to buy it for 1 unit. Can buy > 1 variable but only immediately after seeing it.
- After all N ($=365?$) variables are drawn, the top k ($=5$) values drawn are determined.
- The player pays a penalty of M for each of these k draws they did not purchase.

Where did this game come from?

- GoldCorp plays this game.
- They play with $N = 365$ and $k = 5$.
- They'd rather not play but must.
- The random variable is the peak load for each of 365 days** (simplified)
- Goldcorp pays substantial penalty for the power they generate on the (retrospective) top 5 days.

The Musselwhite Mine

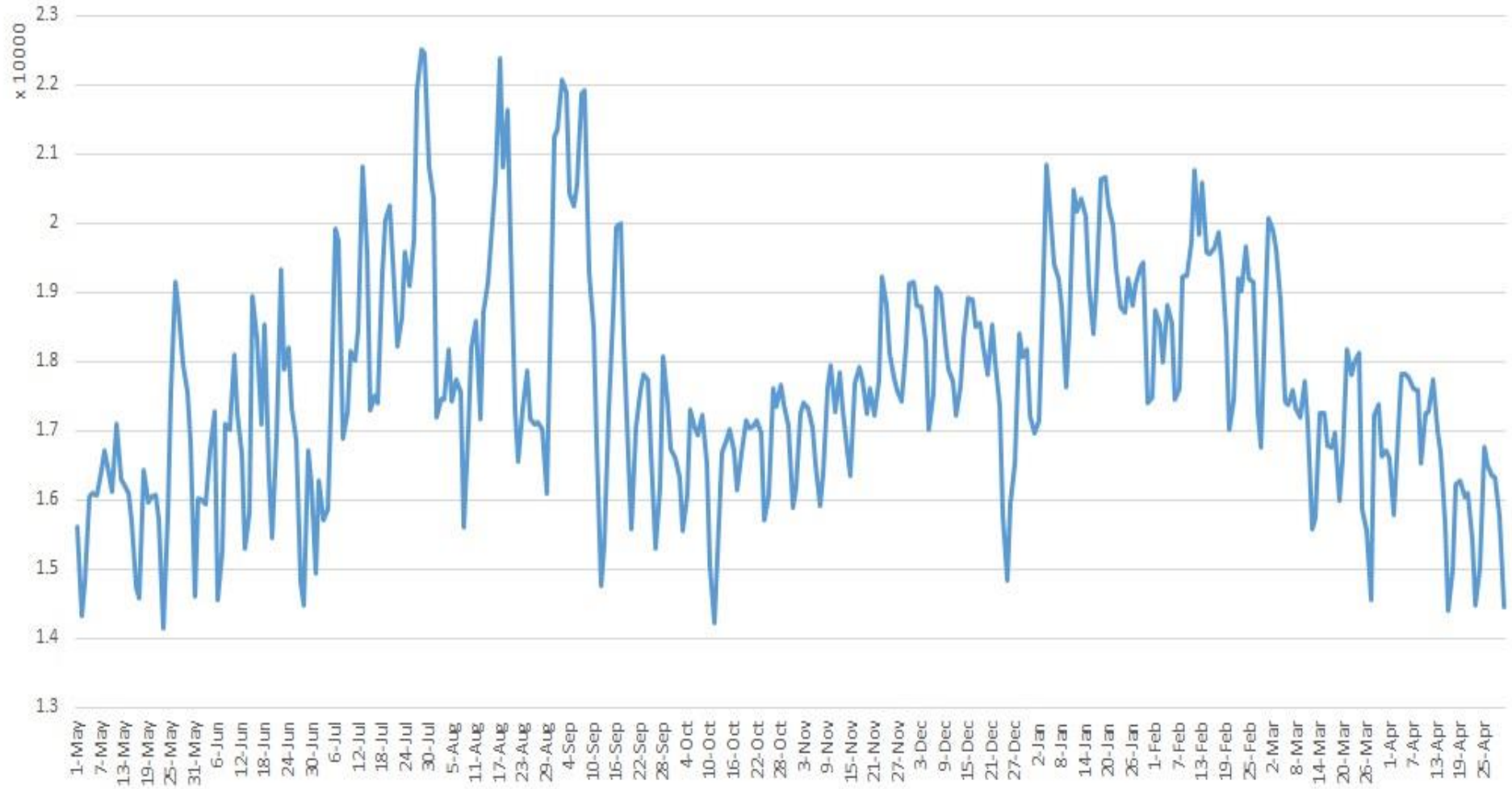
Photo: Goldcorp





- Musselwhite is a fly-in/out underground mine located 500 km north of Thunder Bay. Owned by Goldcorp.
- Musselwhite has produced over 4 million ounces of gold since April 1997. Gold recoveries are 96%.
- Goldcorp is **executing on a roadmap** to increase production and **decrease costs** at its Musselwhite Mine.
- **This is where we come in!!**

2015-2016 daily peak load

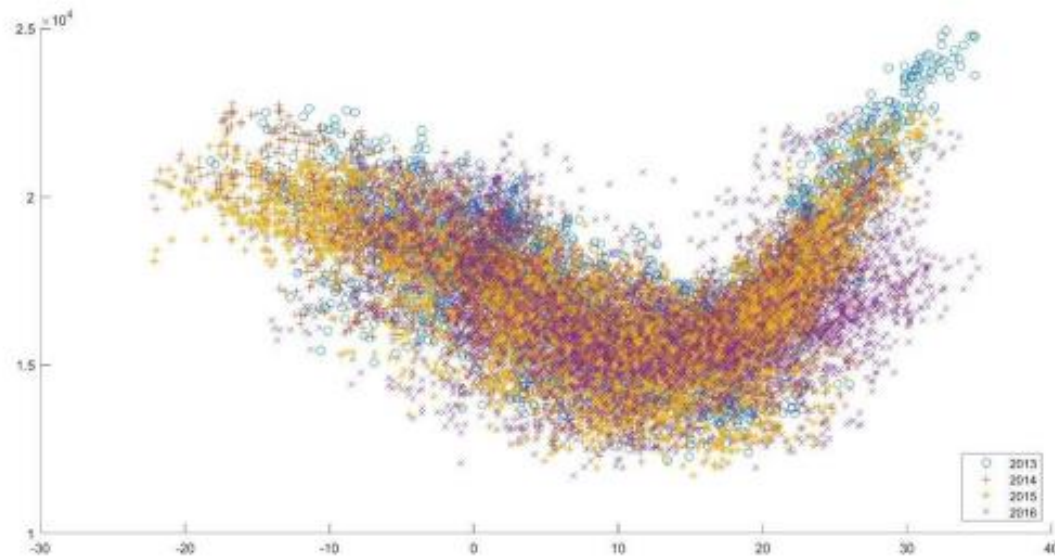


Lots of questions here

- How to play this?
- Does a naïve strategy work?
- Or do we need modelling and analysis?
- From a completely different angle
- WHY is the market run this way? Does it make sense? Could it be run better?

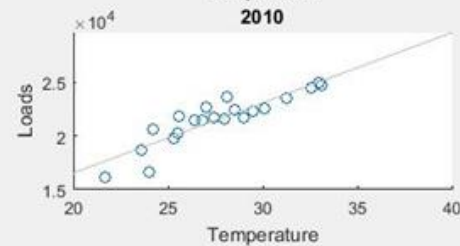
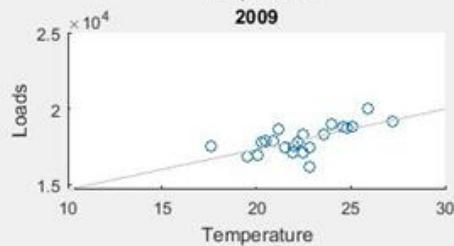
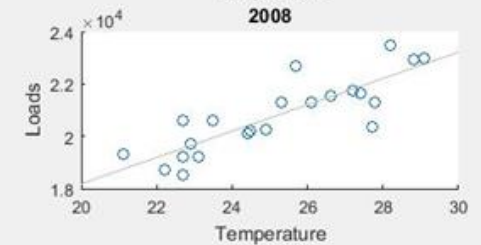
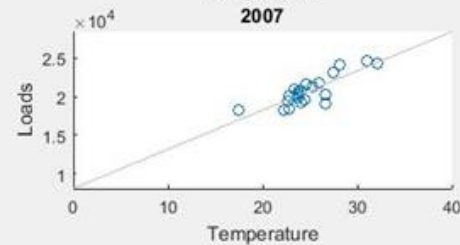
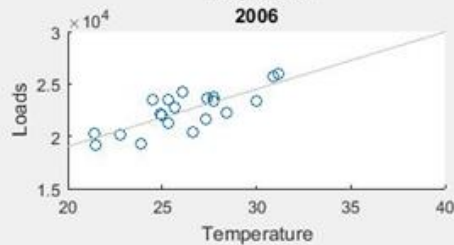
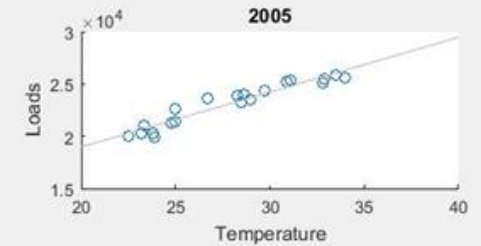
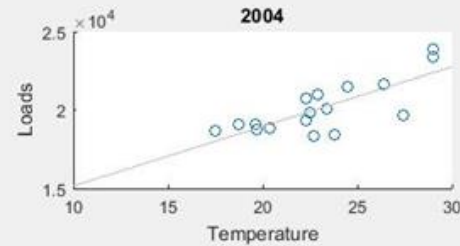
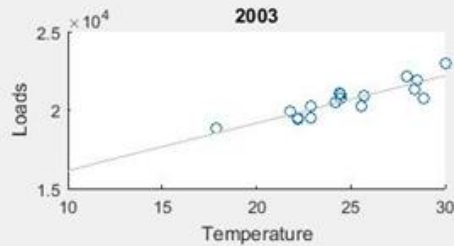
Can we predict this?

Toronto Load vs. Temp
(Load: IESO, Temperature: Environment Canada)

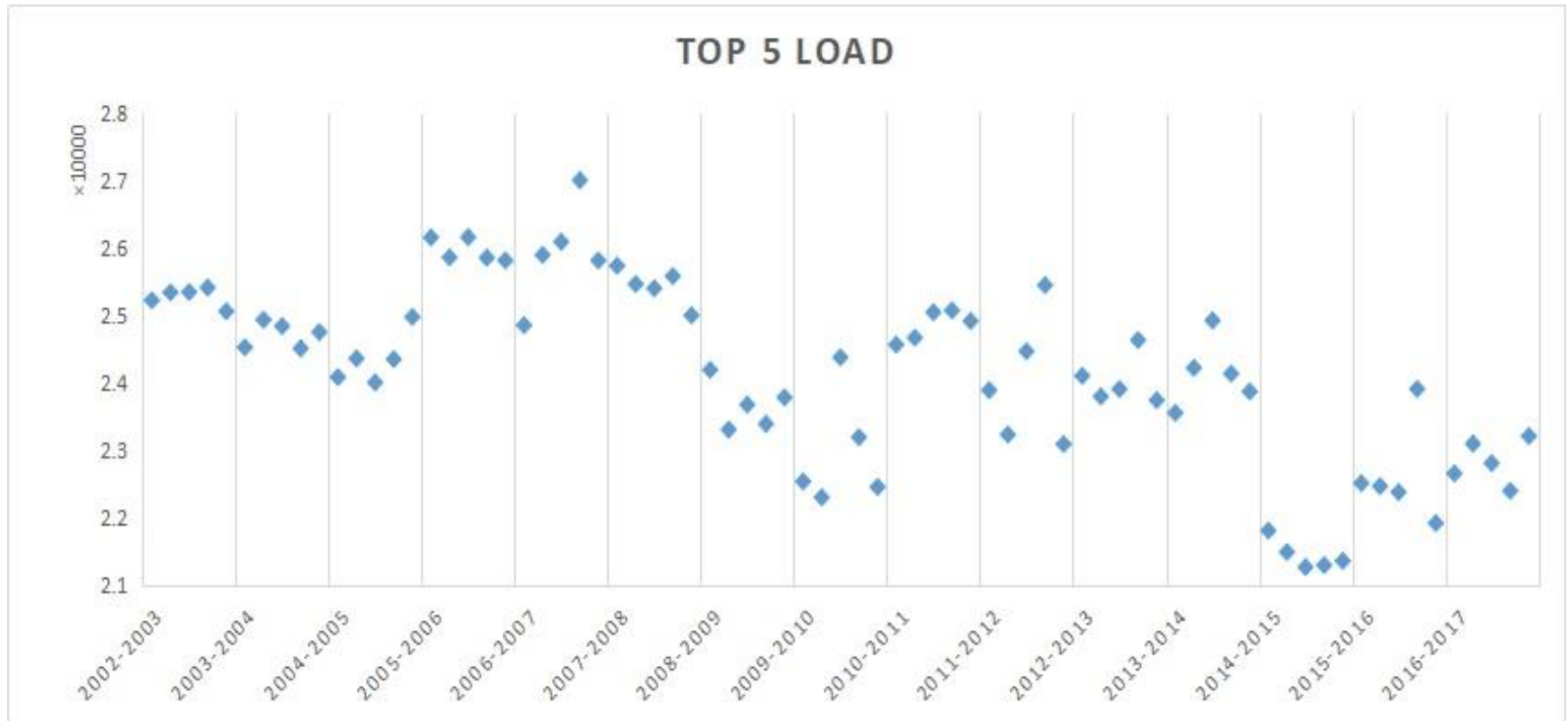


Much better if use typical peak hours

2003-2010 July 4-5pm



Historical Data Analysis



Obvious Naïve Algorithm

- Use the 5th highest realized load from the previous year as a threshold for the next year.
- This strategy is terrible.
- We need more analysis

Yr	03-04	04-05	05-06	06-07	07-08	08-09	09-10
#	2	23	4	5	0	1	24
Yr	10-11	11-12	12-13	13-14	14-15	15-16	10-11
#	9	8	4	0	10	10	9

Simplified game

- In succession, draw N random variables:
 $X_N, X_{N-1}, \dots, X_2, X_1$.
- After each variable is drawn the player can decide whether to buy it for 1 unit.
- You can buy multiple tickets.
- After all N variables are drawn, the maximum value drawn is determined. If the player did not buy it they pay a penalty of M .

Observations

- If we played until we took a draw and then stopped, and were trying to maximize the expected value of the draw we took, that would be the “secretary game” or the “marriage game” which is well studied.
- In continuous time this is a bit related to something called a “Lookback option”
- The game is only interesting if $M < 1$.

More assumptions

- To fix ideas we'll begin with a setting in which all the r.v.s are drawn independently
- From identical $U(0,1)$ r.v.s
- And we can't predict anything
- What is the best way to play this game so as to minimize the expected outlay?
- And what is the expected cost of the game?

Solution Strategy

- Straight dynamic programming not possible
- This one is complicated by needing to keep track of the maximum.
- Still, start backwards with a simple preliminary result

Case I: $N = 2$

- Suppose we are at the last two stages of the game and have not yet drawn x_2 or x_1 .
- We have not yet purchased any draws.
- However, the largest value that has been drawn so far is known to be y .

Structure of solution

- Clearly at the 2nd last draw you won't keep x_2 unless it is at least as large as y .
- Depending on how low y is, you might not even keep some $x_2 > y$
- Optimal solution should be of form:
- Buy the draw if it is larger than some threshold Θ , otherwise wait until the last draw.

Three alternatives

- $0 \leq x_2 \leq y$: Don't buy. At final draw you buy if $x_1 \geq y$, otherwise you pay a penalty M .
Expected cost to go: $yM + (1-y) = 1 + (M-1)y$.
- $y \leq x_2 \leq \Theta$: Don't buy. At final draw you buy if $x_1 \geq x_2$, otherwise pay penalty M .
Cost to go: $x_2M + (1-x_2) = 1 + (M-1)x_2$.
- $\Theta \leq x_2 \leq 1$. Buy. At final draw do nothing if $x_1 \leq x_2$, but buy again if $x_1 > x_2$. No penalty here as peak captured. Cost to go: $1 + (1-x_2)$

Overall cost

- $\int_0^y [1 + (M-1)y]dx + \int_y^\Theta [1 + (M-1)x]dx + \int_\Theta^1 [2 - x] dx = \frac{1}{2}(M-1)y^2 + \frac{1}{2}M\Theta^2 + 3/2 - \Theta$
- Checking critical point $\Theta = 1/M$ against endpoints:
- If $y \leq 1/M \leq 1$ then the optimal policy is to pick $\Theta = 1/M$.
- If $y > 1/M$ the optimal policy is to pick $\Theta = y$.

Corresponding costs to go:

- Case 1: (small max) $0 \leq y \leq 1/M$:
 $V(2 \text{ stages left}) = \frac{1}{2} (M-1)y^2 + 3/2 - (1/2M)$.
- Case 2: (big max) $1/M < y \leq 1$:
 $V(2 \text{ stages left}) = \frac{1}{2} (M-1)y^2 + \frac{1}{2} My^2 + 3/2 - y$
- This obeys “smooth pasting” (continuous, first derivative continuous) at join. Just like American option problem.
- Also note $dV/dy > 0$

Case 2

- Suppose we are at the last two stages of the game and have not yet drawn x_2 or x_1 .
- We have already purchased a draw of z .
- For now, assume that there was no previous draw $> z$, nor any subsequent draw of $> z$ that was not purchased.

Structure of Solution

- At 2nd last draw you won't keep x_2 unless it is at least as large as z .
- Depending on how low z is, you might not even keep it then.
- It seems reasonable that an optimal strategy will continue, as earlier, to be of the form: buy the draw if it is larger than Θ , otherwise wait until the final draw.

Optimal Control/Cost to Go

- $\theta = z + 1/M$ provided $z \leq (M-1)/M$
 $\theta = 1$ provided $(M-1)/M < z \leq 1$.
- Corresponding (2 stage costs to go) are:
- $F(z) = \frac{1}{2} (1-z) (3+z) - 1/(2M)$ $z \leq (M-1)/M$
- $F(z) = \frac{1}{2} (1-z) [1 + z + M(1-z)]$ $1 - 1/M \leq z \leq 1$.
- Smooth pasting still, and $dF/dz < 0$
- Also $dF/dM > 0$, $d^2F/dM^2 < 0$

Can put together for 3 stage game

- Here we buy if $x_3 > \phi$ and don't buy if $x_3 < \phi$.
- In the case when we buy we are in the “z case” and if we don't buy we are in the “y case”.
- Can assume $\phi > 1/M = \theta$

Three stages

- Y case – not bought , but have seen draw of y
- $V_A(y) = \frac{1}{2} (M-1)y^2 + 3/2 - (1/2M)$; $0 \leq y \leq 1/M$:
 $V_B(y) = \frac{1}{2} (M-1)y^2 + \frac{1}{2} My^2 + 3/2 - y$; $1/M < y \leq 1$:
- Z case – bought a draw of z, biggest yet seen:
- $F_A(z) = \frac{1}{2} (1-z) (3+z) - 1/(2M)$; $z \leq (M-1)/M$
 $F_B(z) = \frac{1}{2} (1-z) [1 + z + M(1-z)]$; $(M-1)/M \leq z \leq 1$.
- $\Phi = \text{sqrt}(2M-1)/M$ (if $1 - 1/M < 1$, i.e. $M-1 < M$)

Conclusions for larger game

- Optimal threshold will need to depend on $Y_1, Y_2, Y_3, Y_4, Y_5; Z_1, Z_2, Z_3, Z_4, Z_5$.
- Proceed with Least Squares Monte Carlo on this 10 dimensional space?
- Nonstationarity a bigger issue.

A nice heuristic

- At each date t , $1 < t < 365$, make a “year” based on $C_1, C_2, \dots, C_t, L_{t+1}, \dots, L_{365}$ where C_k is the highest load realized on day k of the current year and L_j the highest load realized on day j of the previous year.
- Use this year to backtest a threshold, and use $\min(\text{this thresh}, \text{fifth lowest load shuttered})$
- This appears to work rather well.

Conclusions

- Some nice math problems arise in industry
- The Ontario electricity market is really messed up
- Financial math works because it is easy to get the data and you can trust it.
- It doesn't work because the rules keep changing.

Thank you!!

Musselwhite Mine (Source: GoldCorp Annual Report, 2016)

Category	Detail
Location	Opapamiskan Lake, Ontario
Type of Mine	Underground
Processing Method	Carbon in pulp recovery
Milling/Processing capacity	4500 tonnes per day
Power Demand: Mine	18MW
Power Demand: Mill	7MW
Employees/Contractors	749
Production (2017 guidance)	265,000 oz
AISC (2017 guidance)	\$715 per oz
Gold Reserves (measured)	310,000 oz
Gold Resources (inferred)	1.17 million oz