

# Department of Mathematics

## Algebraic Number Theory, Math 4151/9051A, Fall 2019

**Class times and location:** MWF at 1:30--2:30pm in MC 108

**Prerequisites:** [Mathematics 4120A/B](#); [Mathematics 3151A/B](#) strongly recommended but not required.

### Overview

This is an introduction to algebraic number theory. Specific topics we will study include number fields and rings, algebraic numbers and integers, prime factorization in integer rings, and class and unit groups of integer rings. Depending on the available time, other potential topics are zeta functions of number fields, counting primes in integer rings, and (very basic) class field theory.

One of the leading contributors to the field was Richard Dedekind, and like many others, he was inspired by Fermat's Last Theorem. The history of the theorem is long and convoluted, and complicated by the fact that it was not proven until 1994, long after his death. As is well known (in number theory), Fermat claimed to have a proof but lacked the space to prove it, and all attempts since then to rediscover his proof have failed. We now believe that his proof assumed certain commutative rings were all principal ideal domains. However, not all relevant rings are PIDs, so the proof would have been flawed. Rather, one needs Dedekind's notion of ideal numbers to recover a suitable notion of prime factorization. We will retrace some of their steps.

### Instruction

**Instructor:** [Chris Hall](#)

**Office Hours:** after class

**Text:** There are two texts for this course:

- I. Stewart and D. Tall, [Algebraic Number Theory and Fermat's Last Theorem](#);

Other nice references include:

D.A. Marcus, [Number Fields](#);

D. Cox, [Primes of the Form  \$x^2+ny^2\$](#) ;

D. Lorenzini, [An Invitation to Arithmetic Geometry](#);

G.J. Janusz, [Algebraic Number Fields](#);

S. Lang, [Algebraic Number Theory](#);

J. Neukirch, [Algebraic Number Theory](#).

These are of varying levels of difficulty, so browse a few before choosing one. (None is required though!)

### Expectations

**Dropbox:** I will share electronic articles and notes using [Dropbox](#). Please [provide me](#) with the e-mail address you use to [login](#).

**Attendance:** Our class is small and someone's absence can greatly impact the rest of the class. Therefore you are expected to attend class or to let me know in advance when you are unable to attend.

**Exercises:** There will be exercises sets assigned throughout the semester, six to eight. I will post the list of exercises on OWL with a due date (typically one week after the assignment opens). You should typeset your answers and submit an electronic version via [TBD](#).

**Projects:** Each person must schedule a 60-minute meeting with me during the exam period Dec 8--19 for a private oral exam. Please complete [a survey](#) to indicate what times would work for you; I will be the only one who sees your answers, and I will use it during the last week of class to schedule the exams. If you need me to commit before Dec 8 to the date of your exam, please let me know asap.

To prepare for this exam you must:

1. select a topic from my list or negotiate your own topic, and obtain my approval by Nov 1;
2. read material I suggest to you;
3. prepare an outline for a 5-page document relevant to your topic, and obtain my approval by Nov 15;
4. use your outline to complete your document;
5. bring a printed copy of the latest version to your final exam.

You should certainly meet with me if you have questions or concerns about how to prepare. At the exam in December we will discuss your project and analyze your document.

---

## Evaluation

**Basis:** Your performance will be measured using two tangible items: exercises and a project. They will contribute 60% and 30% to respectively your final grade. The remaining 10% consists of a grade for the oral exam.

**Scholastic offences:** Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site:

[http://www.uwo.ca/univsec/handbook/appeals/scholastic\\_discipline\\_grad.pdf](http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_grad.pdf)

---

## Course Policies:

**Senate Policy on Prerequisites:** Prerequisite checking is the student's responsibility. Unless you have either the requisites for this course or written special permission from your Dean to enroll in it, you may be removed from this course and it will be deleted from your record. This decision may not be appealed. You will receive no adjustment to your fees in the event that you are dropped from a course for failing to have the necessary prerequisites.

**Course Websites:** Students should check OWL (<http://owl.uwo.ca>) or the course website (if not hosted on OWL) on a regular basis for news and updates for all of the courses in which they are enrolled. This is the primary method by which information will be disseminated to all students in each class. Students are responsible for checking OWL on a regular basis.

**Email:** Students must use their Western (@uwo.ca) email addresses when contacting their instructors.

**Scholastic Offences:** Scholastic Offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web sites:

[http://www.uwo.ca/univsec/handbook/appeals/scholastic\\_discipline\\_undergrad.pdf](http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_undergrad.pdf)

[http://www.uwo.ca/univsec/handbook/appeals/scholastic\\_discipline\\_grad.pdf](http://www.uwo.ca/univsec/handbook/appeals/scholastic_discipline_grad.pdf)

**Absence/Missed work:** If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to your Dean's office *as soon as possible* and contact your instructor *immediately*. It is the student's responsibility to make alternative arrangements with his or her instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office *immediately*. For further information concerning UWO's Policy on Accommodation for Medical Illness please refer to this policy at:

[https://www.uwo.ca/univsec/handbook/appeals/accommodation\\_medical.pdf](https://www.uwo.ca/univsec/handbook/appeals/accommodation_medical.pdf)

A student requiring academic accommodation due to illness should use the Student Medical Certificate when visiting an off-campus medical facility or request a "Record's Release Form" (located in the Dean's Office) for visits to Student Health Services. The form can be found here:

[https://www.uwo.ca/univsec/handbook/appeals/medicalform\\_15JUN.pdf](https://www.uwo.ca/univsec/handbook/appeals/medicalform_15JUN.pdf)

**Make-up exams and conflicts:** If you know ahead of time that you are unable to attend a midterm or final exam, you must let your instructor know at least *two weeks in advance* so alternative arrangements can be made. See also the [University's policy on final exam conflicts](#).

**Accommodation and accessibility:** If you are unable to meet a course requirement due to illness or other serious circumstances, you must seek approval for the absence as soon as possible. Approval can be granted either through a self-reporting of absence or via the Dean's Office/Academic

Counselling unit of your Home Faculty. If you are a Science student, the Academic Counselling Office of the Faculty of Science is located in NCB 280, and can be contacted at [scibmsac@uwo.ca](mailto:scibmsac@uwo.ca).

For further information, please consult the university's policy on academic consideration for student absences: [https://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/Academic\\_Consideration\\_for\\_absences.pdf](https://www.uwo.ca/univsec/pdf/academic_policies/appeals/Academic_Consideration_for_absences.pdf).

If you miss the Final Exam, please contact your faculty's Academic Counselling Office as soon as you are able to do so. They will assess your eligibility to write the Special Exam (the name given by the university to a makeup Final Exam).

You may also be eligible to write the Special Exam if you are in a "Multiple Exam Situation" (see [http://www.registrar.uwo.ca/examinations/exam\\_schedule.html](http://www.registrar.uwo.ca/examinations/exam_schedule.html)).

**Academic Policies:** The website for Registrarial Services is <http://www.registrar.uwo.ca>.

In accordance with policy, <http://www.uwo.ca/its/identity/activatenonstudent.html>, the centrally administered e-mail account provided to students will be considered the individual's official university e-mail address. It is the responsibility of the account holder to ensure that e-mail received from the University at his/her official university address is attended to in a timely manner.

Electronic devices (including cell phones and calculators) are NOT allowed on exams and may be confiscated.

**Support Services:** Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. You may also wish to contact Student Accessibility Services (SAS) at 661-2147 if you have any questions regarding accommodations.

The policy on Accommodation for Students with Disabilities can be found here: [https://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/Academic%20Accommodation\\_disabilities.pdf](https://www.uwo.ca/univsec/pdf/academic_policies/appeals/Academic%20Accommodation_disabilities.pdf).

The policy on Accommodation for Religious Holidays can be found here: [http://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/accommodation\\_religious.pdf](http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_religious.pdf).

Learning-skills counsellors at the Student Development Centre (<http://www.sdc.uwo.ca>) are ready to help you improve your learning skills. They offer presentations on strategies for improving time management, multiple-choice exam preparation/writing, textbook reading, and more. Individual support is offered throughout the Fall/Winter terms in the drop-in Learning Help Centre, and year-round through individual counselling.

Students who are in emotional/mental distress should refer to Mental Health@Western ([http://www.health.uwo.ca/mental\\_health](http://www.health.uwo.ca/mental_health)) for a complete list of options about how to obtain help.

Additional student-run support services are offered by the USC, <http://westernusc.ca/services>.

**Acknowledgment of the Science Student Donation Fund:** Mathematics undergrad courses are supported by the Science Student Donation Fund. If you are a BSc or BMSc student registered in the Faculty of Science or Schulich School of Medicine and Dentistry, you pay the Science Student Donation Fee. This fee contributes to the Science Student Donation Fund, which is administered by the Science Students' Council (SSC). One or more grants from the Fund have allowed for the purchase of equipment integral to teaching this course. You may opt out of the Fee by the end of September of each academic year by completing the online form linked from the Faculty of Science's Academic Counselling site. For further information on the process of awarding grants from the Fund or how these grants have benefitted undergraduate education in this course, consult the Chair of the Department or email the Science Students' Council at [ssc@uwo.ca](mailto:ssc@uwo.ca).

#### © 1878 - 2020 Western University

The Department of Mathematics  
Middlesex College  
London, Ontario, Canada, N6A 5B7  
Tel: 519.661.3639  
[math-inquiry@uwo.ca](mailto:math-inquiry@uwo.ca) (undergraduate), [math-grad-program@uwo.ca](mailto:math-grad-program@uwo.ca)  
(grad), [mathdept@uwo.ca](mailto:mathdept@uwo.ca) (other)  
[Privacy](#) | [Web Standards](#) | [Terms of Use](#) | [Accessibility](#)

#### Key Topics:

About Us  
Undergraduate  
Graduate  
Research  
People

#### Popular Resources:

Calculus 1000A/B online  
Office of the Registrar  
Academic Calendar  
Intranet Login

