

The University of Western Ontario
Department of Applied Mathematics
Course Outline
Applied Mathematics 4617b/AM9566: Numerical Solution of PDE's

Description: Finite difference methods, stability analysis for time-dependent problems.

Prerequisite(s): permission of the instructor + an ability to program in a compiled language.

Book: "Finite Difference Schemes and Partial Differential Equations", John C. Strikwerda, Second edition, SIAM, 2004 (required)

Instructor: Dr. A. MacIsaac (allanb@uwo.ca), Middlesex College, Room 285

Lectures: Monday, Wednesday, Friday 1:30pm - 2:20pm UCC 65

Office Hours: Tuesday 2:30pm to 5:00pm

Note this course has joint lectures for AM4617 and AM9566, however the Graduate student evaluation will include unique assignments and exams and may require additional reading on topics not covered in class.

Marking Scheme:

40% Assignments/quizzes (up to 8 assignments/quizzes. Biweekly)

25% Midterm Exam (There is no makeup exam. Marks will be reassigned to final exam)

35% Final Exam

Learning Outcomes:

1. Derive finite difference schemes using Taylor series.
2. Determine the consistency of a difference scheme.
3. Explain the proper function spaces and discrete norms for grid functions for use in analysis of stability.
4. Establish the stability of a difference scheme using (1) Heuristic approach (2) Energy method (3) von Neumann method (4) Matrix method.
5. Recall the CFL condition its relation with stability.

6. Explain the convergence of the finite difference approximations and its relation with consistency and stability via Lax theorem;
7. Determine the order of accuracy of a finite difference scheme.
8. Implement finite difference schemes on computers and perform numerical studies of the stability and convergence properties of the schemes.
9. Explain the role and the control of numerical diffusion and dispersion in computation; to determine how numerical phase speed and group velocity may deviate from the theoretical phase speed and group velocity and the numerical techniques to handle such issues.
10. Recall numerical methods that efficiently handle a multidimensional problem.
11. Recall alternating direction methods that reduce higher dimensional problems into a sequence of one dimensional problems.
12. Recall the maximum principles for numerical schemes for Laplace equations.
13. Recall iterative techniques for solving the linear systems resulting from finite difference or finite element discretization.

We will try to cover the following chapters at various depths.

Chapter 1: Hyperbolic Partial Differential Equations.

1. Overview of Hyperbolic Partial Differential Equations.
2. Boundary Conditions.
3. Introduction to Finite Difference Schemes.
4. Convergence and Consistency.
5. Stability.
6. The Courant-Friedrichs-Lewy Condition.

Chapter 2: Analysis of Finite Difference Schemes.

1. Fourier Analysis.
2. Von Neumann Analysis.
3. Comments on Instability and Stability

Chapter 3: Order of Accuracy of Finite Difference Schemes.

1. Order of Accuracy.
2. Stability of Lax-Wendroff and Crank-Nicolson Schemes.
3. Difference Notation and Difference Calculus.
4. Boundary Conditions for Finite Difference Schemes.
5. Solving Tridiagonal Systems.

Chapter 4: Stability for Multistep Schemes.

1. Stability for the Leapfrog Scheme.
2. Stability for General Multistep Schemes.

Chapter 5: Dissipation and Dispersion.

1. Dissipation.

2. Dispersion.
3. Group Velocity and the Propagation of Wave Packets.

Chapter 6: Parabolic Partial Differential Equations.

1. Overview of Parabolic Differential Equations.
2. Parabolic Systems and Boundary Conditions.
3. Finite Difference Schemes for Parabolic Equations.
4. The Convection-Diffusion equation.
5. Variable Coefficients

Chapter 7: Systems of Partial Differential Equations.

1. Stability of Finite Difference Schemes for Systems.
2. Finite Difference Schemes in Two and Three Dimensions.
3. The Alternating Direction Implicit Method.

Chapter 10: Convergence Estimates for Initial Value Problems.

1. Convergence Estimates for Smooth Initial Functions.
2. Related Topics.
3. Convergence Estimates for Non-smooth Initial Functions.
4. Convergence Estimates for Parabolic Differential Equations.
5. The Lax-Richtmyer Equivalence Theorem.
6. Analysis of Multistep Schemes.
7. Convergence Estimates for Second-Order Differential Equations.

Chapter 12: Elliptic Partial Differential Equations and Difference Schemes.

1. Overview of Elliptic Partial Differential Equations.
2. Regularity Estimates for Elliptic Equations.
3. Maximum Principles.
4. Boundary Conditions for Elliptic Equations.
5. Finite Difference Schemes for Poisson's Equations.
6. Coordinate Changes and Finite Differences.

Chapter 13: Linear Iterative Methods.

1. Solving Finite Difference Schemes for Laplace's Equation in a Rectangle.
2. Eigenvalues of the Discrete Laplacian
3. Analysis of the Jacobi and Gauss-Seidel Methods.
4. Convergence Analysis of Point SOR.
5. Consistently Ordered Matrices.
6. Linear Iterative Methods for Symmetric, Positive Definite Matrices.
7. The Neumann Boundary Value Problem.

Chapter 14: The Method of Steepest Descent and the Conjugate Gradient Method.

1. The Method of Steepest Descent.
2. The Conjugate Gradient Method.
3. Implementing the Conjugate Gradient Method.
4. A convergence Estimate for the Conjugate Gradient Method.
5. The Preconditioned Conjugate Gradient Method.

Accommodation and Accessibility: If you are unable to meet a course requirement due to illness or other serious circumstances, you must seek approval for the absence as soon as possible. Approval can be granted either through a self-reporting of absence or via the Dean's Office/Academic Counselling unit of your Home Faculty. If you are a Social Science student, the Academic Counselling Office of the Faculty of Social Science is located in SSC 2105, and can be contacted at ssaco@uwo.ca. If you are a Science student, the Academic Counselling Office of the Faculty of Science is located in NCB 280, and can be contacted at scibmsac@uwo.ca. For further information, please consult the university's policy on academic consideration for student absences.

Academic Policies: The website for Registrarial Services is <http://www.registrar.uwo.ca>. In accordance with policy, the centrally administered e-mail account provided to students will be considered the individual's official university email address. It is the responsibility of the account holder to ensure that email received from the University at his/her official university address is attended to in a timely manner.

Scholastic offences are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offence, at the following Web site: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_undergrad.pdf. We will clearly indicate the level of acceptable collaboration on assignments and projects. All assignments and papers may be checked for textual similarity for detection of plagiarism. All papers submitted for such checking will be included as source documents in the reference database for the purpose of detecting plagiarism of papers subsequently submitted to the system. Use of the service for papers is subject to the licensing agreement currently between The University of Western Ontario and Turnitin.com (<http://www.turnitin.com>). Programming assignments may be checked for similarity using MOSS (Measure of Software Similarity).

Support Services: Please contact the course instructor if you require lecture or printed material in an alternate format or if any other arrangements can make this course more accessible to you. You may also wish to contact Student Accessibility Services (SAS) at 661-2147 if you have any questions regarding accommodations. The policy on Accommodation for Students with Disabilities can be found here: https://www.uwo.ca/univsec/pdf/academic_policies/appeals/Academic%20Accommodation_disabilities.pdf The policy on Accommodation for Religious Holidays can be found here: http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_religious.pdf Learning-skills counsellors at the Student Development Centre (<http://www.sdc.uwo.ca>) are ready to help you improve your learning skills. They offer presentations on strategies for improving time management, multiple-choice exam preparation/writing, textbook reading, and more. Individual support is offered throughout the Fall/Winter terms in the drop-in Learning HelpCentre, and year-round through individual counselling. Students who are in emotional/mental distress should refer to Mental Health@Western (http://www.health.uwo.ca/mental_health) for a complete list of options about how to obtain help. Additional student-run support services are offered by the USC, <http://westernusc.ca/services>.