Math Club at Western (MaCAW) 1st Annual Math Contest
23 January 2018

- This contest consists of 4 questions worth 10 marks each, for a total of 40 marks. The questions in this contest are arranged in order of increasing difficulty.
- The time limit for this contest is **1 hour**.
- A **full solution** is required for each question. Marks are awarded for completeness and clarity.
- When applicable, express all answers as **simplified exact numbers**; for example, use $\pi + 2$ instead of 5.1415... , and use $1 - \sqrt{2}$ instead of –0.4142... .
- Complete **all** solutions on the lined paper provided. Write your name on the top of **every** page being submitted for grading. The lined paper provided may also be used for rough work. Do **not** include solutions and rough work on the same page.
- **No** materials are permitted during the contest other than writing materials.
- **No** washroom breaks are permitted.
- If you have any questions, **raise your hand**. Do **not** go to the front of the room.
- If you finish the contest early, **raise your hand**. Your solutions will be collected from you. **Quietly** collect your belongings and exit the room.

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1. In triangle ΔABC, $|AB| = 14$, $|BC| = 12$, $|AC| = 10$, and D is a point on the line segment AC such that $|AD| = 4$. If E is a point on the line segment BC such that $Area(\Delta ABC) = 2 \cdot Area(\Delta CDE)$, what is the area of triangle ΔABE?

2. Determine all functions $f : \mathbb{R}^+ \to \mathbb{R}$ for which $f(1) = 1$ and $f(x^2 y^2) = f(x^4 + y^4)$ for all positive real numbers x and y.

3. Consider the $2 \times n$ chessboard, for positive integers n. Suppose k coins (where k is a nonnegative integer not exceeding $2n$), all showing heads, are placed consecutively on the chessboard from left to right, starting at the upper-left corner and moving to the lower row if necessary.
   Next, we place coins showing tails, one by one, in the empty squares of the chessboard in any order we please. Immediately after placing each of these coins, we flip each of the coins in the adjacent squares (where two squares are said to be adjacent if and only if they share at least one corner). Determine, as a function of n, the number of values of k for which it is possible to place the coins so that once all $2n$ of the squares are occupied, all $2n$ of the coins show tails.

4. For each positive integer n, we define $P(n) = n(n + 1)(2n + 1)(3n + 1) \ldots (16n + 1)$. Find the greatest common divisor of $P(1), P(2), P(3), \ldots, P(1000)$.