

The University of Western Ontario  
Department of Mathematics

**Mathematics 9607B, Winter 2019**  
**COMPLEX ANALYTIC GEOMETRY**

**Lectures:** Mon, Wed, 11:00am–12:30pm, MC 107

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**Course Website:** <http://uwo.ca/math/faculty/adamus/teaching/9607B2019/>

**Material outline:** Given the immensity of the subject, this course will by no means cover (or even touch) all important areas of the analytic geometry. Any attempt on doing so would inevitably result in an overloaded inventory of pairwise unrelated “well-known” facts. Instead, I choose not to deny my personal bias, and customize the choice of material so that it leads to and gives a good feeling of the local geometry of analytic mappings, which is one of the interests of current research. The following is the list of main topics to be covered:

- Analytic sets and germs - definitions and basic topological properties; dimension.
- Regular and singular loci.
- Hironaka’s division algorithm.
- Weierstrass Preparation, Division and Finiteness theorems.
- Rings of germs of holomorphic functions.
- Proper projections: Remmert Proper Mapping theorem, and the local description of analytic sets.
- Analyticity of the singular locus, irreducibility and irreducible components.
- Coherence theorems of Oka and Cartan.
- Coherent sheaves and complex analytic spaces.
- Remmert Rank and Open Mapping theorems.
- Analytic tensor product and fibred product of analytic mappings.
- Hironaka flatness criterion and its recent applications.
- Effective criteria for regularity of analytic mappings (time permitting).

**Problem Sets / Evaluation:** The course mark will be based on 4-5 bi-weekly homework assignments, class participation, and a final presentation. Class attendance is required.

**Prerequisites:** No particular prerequisites are required, other than a good command of undergraduate mathematics. However, having heard of the following could not hurt: a first course in commutative algebra, basic theory of smooth manifolds, basic facts from several complex variables. (All necessary facts will be recalled as they appear relevant.)

**Readings:** There will be no textbook for this course. Instead, the complete set of detailed (and recently revised) lecture notes will be available through the course web site. The following readings are recommended only for further study on certain topics covered:

1. S. Abhyankar, “Local analytic geometry”, Vol. XIV Academic Press, New York-London, 1964.
2. J. Adamus, *Flatness testing and torsion freeness of analytic tensor powers*, J. Algebra **289** (2005), 148–160.
3. J. Adamus, E. Bierstone and P. D. Milman, *Geometric Auslander criterion for flatness*, Amer. J. Math. **135** (2013), 125–142.
4. J. Adamus, E. Bierstone and P. D. Milman, *Geometric Auslander criterion for openness of an algebraic morphism*, Bull. Lond. Math. Soc. **45** (2013), 1060–1064.
5. J. Adamus and H. Seyedinejad, *A fast flatness testing criterion in characteristic zero*, Proc. Amer. Math. Soc. **143** (2015), 2559–2570.
6. M. Auslander, *Modules over unramified regular local rings*, Illinois J. Math. **5** (1961), 631–647.
7. E. Bierstone and P. D. Milman, “The local geometry of analytic mappings”, Dottorato di Ricerca in Matematica, ETS Editrice, Pisa, 1988.
8. H. Cartan, *Idéaux et modules de fonctions analytiques de variables complexes*, Bull. Soc. Math. France **78** (1950), 29–64.
9. E. M. Chirka, “Complex analytic sets”, **46**. Kluwer Academic Publishers Group, Dordrecht, 1989.
10. D. Eisenbud, “Commutative Algebra with a View Toward Algebraic Geometry”, Springer, New York, 1995.
11. G. Fischer, “Complex Analytic Geometry”, Lecture Notes in Mathematics, **538**. Springer, Berlin-New York, 1976.
12. A. Galligo and M. Kwieciński, *Flatness and fibred powers over smooth varieties*, J. Algebra **232** (2000), 48–63.
13. H. Grauert and R. Remmert, “Analytische Stellenalgebren”, Springer, Berlin-New York, 1971.
14. H. Hironaka, *Stratification and flatness*, in “Real and Complex Singularities”, Proc. Oslo 1976, ed. Per Holm, Stijthof and Noordhof (1977), 199–265.
15. E. Kunz, “Introduction to Commutative Algebra and Algebraic Geometry”, Birkhäuser, Boston, 1985.
16. S. Łojasiewicz, “Introduction to Complex Analytic Geometry”, Birkhäuser, Basel, 1991.
17. J.-P. Serre, *Faisceaux algébriques cohérents*, Ann. of Math. **61** (1955), 197–278.
18. H. Whitney, “Complex analytic varieties”, Addison-Wesley, Reading, Mass.-London-Don Mills, Ont., 1972.

**Statement on Academic Offenses:** Scholastic offenses are taken seriously and students are directed to read the appropriate policy, specifically, the definition of what constitutes a Scholastic Offense, at the following Web site: [http://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/scholastic\\_discipline\\_grad.pdf](http://www.uwo.ca/univsec/pdf/academic_policies/appeals/scholastic_discipline_grad.pdf)

**Medical Excuse Regulations:** Students who are unable to meet a course requirement worth 10% or more of the course grade, due to illness or other serious circumstances, must provide valid medical or other supporting documentation to their home faculty Dean's Office as soon as possible and contact their instructor immediately. It is the student's responsibility to make alternative arrangements with their instructor once the accommodation has been approved and the instructor has been informed. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office as soon as possible.

The Policy on Accommodation for Medical Illness is available at

[http://www.uwo.ca/univsec/pdf/academic\\_policies/appeals/accommodation\\_medical.pdf](http://www.uwo.ca/univsec/pdf/academic_policies/appeals/accommodation_medical.pdf)

A student requiring academic accommodation due to illness should use the Student Medical Certificate when visiting an off-campus medical facility or request a Record's Release Form (located in the Dean's Office) for visits to Student Health Services. The form can be found here:

[http://studentservices.uwo.ca/secure/medical\\_document.pdf](http://studentservices.uwo.ca/secure/medical_document.pdf)

**Mental Health Statement:** Students who are in emotional/mental distress should refer to Mental Health@Western [http://www.health.uwo.ca/mental\\_health/](http://www.health.uwo.ca/mental_health/) for a complete list of options about how to obtain help.

**Accessibility Statement:** Please contact the course instructor if you require material in an alternate format or if you require any other arrangements to make this course more accessible to you. You may also wish to contact Services for Students with Disabilities (SSD) at 661-2111 ext. 82147 for any specific questions regarding an accommodation.

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Graduate students with disabilities (for example, chronic illnesses, mental health conditions, mobility impairments) are encouraged to register with Student Accessibility Services, a confidential service designed to support graduate and undergraduate students through their academic program. With the appropriate documentation, the student will work with both SAS and their graduate programs (normally their Graduate Chair and/or Course instructor) to ensure that appropriate academic accommodations to program requirements are arranged. These accommodations include individual counselling, alternative formatted literature, accessible campus transportation, learning strategy instruction, writing exams and assistive technology instruction. See <http://www.sdc.uwo.ca/ssd/>.