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Erratum to "Traveling Wave Fronts of Reaction-Diffusion Systems with Delays" *[J. Dynam. Diff. Eq. 13, 651, 687 (2001)]*

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In [1], we combined the idea of upper and lower solutions with a monotone iteration scheme to establish the existence of traveling wave fronts in reaction-diffusion systems with delays. It was discovered recently that our original requirements for upper and lower solutions can not guarantee the monotonicity of the iteration. This is explained below.

The definitions of upper and lower solutions of (2.3) in [1] are

Definition 3.2. A continuous function $\rho:\mathcal{R}\to\mathcal{R}^n$ is called an upper solution of (2.3) if ρ' and ρ'' exist almost everywhere and they are essentially bounded on $\mathcal{R}$, and if ρ satisfies

$$D\rho''(t)-c\rho'(t)+f_c(\rho_t)\leq 0, \quad a.e. \quad \text{on } \mathcal{R}. \tag{1}$$

A lower solution of (2.3) is defined in a similar way by reversing the inequality in (3.3).

In the proof of (iii) of Lemma 3.3 in [1], $w_i(t)=x_{1i}(t)-\rho_i(t)$ was introduced which satisfies (3.8). By (3.6) and the continuity of $\bar{\rho}$, we know that x_{1i} is C^1. Now if $\bar{\rho}(t)$ is an upper solution as is defined in Definition 3.2, then it may not be in C^1 and hence w_{1i} may not be in C^1. When $\bar{\rho}(t)$

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is in C^1, so is w_{1i} and hence, (3.9) holds and the argument following (3.9) in [1] applies. However, when $\bar{\rho}(t)$ is not in C^1, nor is w_{1i} and thus, (3.9) may not be true as piecewise smooth w_{1i} is possible. For example, instead of (3.9), one can not exclude

$$w_i(t)=\begin{cases} ce^{\lambda_{2i}t}+\frac{1}{d_i(\lambda_{2i}-\lambda_{1i})}\left[\int_{-\infty}^t e^{\lambda 1i(t-s)}r_i(s)\,ds+\int_t^\infty e^{\lambda 2i(t-s)}r_i(s)\,ds\right], \\ \text{for } t\le 0; \\ ce^{\lambda_{1i}t}+\frac{1}{d_i(\lambda_{2i}-\lambda_{1i})}\left[\int_{-\infty}^t e^{\lambda 1i(t-s)}r_i(s)\,ds+\int_t^\infty e^{\lambda 2i(t-s)}r_i(s)\,ds\right], \\ \text{for } t\ge 0. \end{cases}$$

for constant $c\neq 0$. In such a case, one can not claim $x_1(t)\le\bar{\rho}(t)$ for $t\in\mathcal{R}$.

Based on the above observation, we need to pose some extra conditions on the upper and lower solutions at those points where smoothness is not satisfied (see (H4) below) so that the main theorems (Theorems 3.6 and Theorems 4.5) in [1] remain valid. We state the modified versions of these two theorems below.

Theorem 3.6′. *Assume that (A1) and (A2) hold. Suppose that (2.3) has an upper solution $\bar{\phi}\in\Gamma$ and a lower solution $\underline{\phi}$ (which is not necessarily in Γ) satisfying (H1)–(H2) and*

(H4) $\sup_{s\le t}\underline{\phi}(t)\le\bar{\phi}(t)$; $\bar{\phi}'(t+)\le\bar{\phi}'(t-)$ and $\underline{\phi}'(t+)\ge\underline{\phi}'(t-)$ for $t\in\mathcal{R}$.

Then (2.3) and (2.11) have a solution. That is, (2.1) has a traveling wave front solution.

Theorem 4.5′. *Assume that (A1) and (A2)* hold. Suppose that (2.3) has an upper solution $\bar{\phi}\in\Gamma^*$ and a lower solution $\underline{\phi}$ (which is not necessarily in Γ^*) satisfying (H1)*–(H3)* and (H4). Then (2.1) has a traveling wave front with $c>1-\min\{\beta_i d_i; i=1,\ldots,n\}$.*

The validity of these theorems can be verified by the smoothening procedure used in [2]. We only give an argument for Theorem 3.6′ since the proof of Theorem 4.5′ is similar. For convenience, we denote by F the operator on the right hand side of (3.6) in [1], that is, $F:C(\mathcal{R};\mathcal{R}^n)\to C(\mathcal{R};\mathcal{R}^n)$ is defined by $F(\phi)=(F_1(\phi),\ldots,F_n(\phi))$ where

$$F_i(\phi)=\frac{1}{d_i(\lambda_{2i}-\lambda_{1i})}\left[\int_{-\infty}^t e^{\lambda 1i(t-s)}H_i(\phi)(s)\,ds+\int_t^\infty e^{\lambda 2i(t-s)}H_i(\phi)(s)\,ds\right].$$

Here, $H:C(\mathcal{R};\mathcal{R}^n)\to C(\mathcal{R};\mathcal{R}^n)$ is defined by (3.2) in [1]. Let $\bar{\rho}(t)=F(\bar{\phi})(t)$ and $\underline{\rho}(t)=F(\underline{\phi})(t)$. By Lemmas 2.5 and 2.6 in [2], we know that $\bar{\rho}$ and $\underline{\rho}$ are a pair of upper and lower solutions of (2.3) which not only satisfy

(H1) and (H2) but are in C^2 now. After this, the proof of Theorem 3.6 in [1] carries over, showing the validity of Theorem 3.6′.

We point out that the same idea was also used in a recent paper [3]. Also, in a more recent preprint [4], instead of allowing non-smooth upper and lower solutions but posing extra condition on their non-smooth points, the authors directly define upper and lower solutions to be in C^1.

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