Problem Set 5

due: June 16

1. Recommended practice problems from Sections 3.9, Chapter 3 Review, 4.1, and 4.1–4.4.

2. Find the following limits:

(a) $\lim\limits_{x\to\infty} \dfrac{e^x}{x^{\ln x}}$

(b) $\lim\limits_{x\to 0^+} x^x$

(c) $\lim\limits_{x\to 1^-} (1-x)\tan(\dfrac{\pi x}{2})$

(d) $\lim\limits_{x\to 1} \left(\dfrac{\sin x}{\sin 1}\right)^{\frac{1}{x-1}}$

(e) $\lim\limits_{x\to 0} (\dfrac{1}{x} - \dfrac{1}{e^x - 1})$.

3. Find derivatives of the following functions:

(a) $f(x) = x^{e^x}$

(b) $g(x) = \left(\dfrac{\sin(mx)}{\sin(nx)}\right)^{(mx)^{nx}}$.

[Hint: Use logarithmic differentiation.]

4. Sketch the graphs of the following functions. Justify your answer.

(a) $f(x) = \ln(x + \sqrt{x^2+1})$

(b) $g(x) = (1+x^2)\cdot e^{-x^2}$.