Problem Set 3

due: June 2

1. Recommended practice problems from Sections 2.3, 2.5, 2.6 and 2.7.

2. Use the Squeeze Theorem to find the following limits. Justify your answers.

(a) $\lim\limits_{x\to 1^-} \sqrt{1-x^2}\cdot \sin^5(\ln(1-x^2))$;

(b) $\lim\limits_{x\to\infty} \sin\left(\dfrac{\pi}{x}\right)\cdot e^{\cos(\pi x)}$;

(c) $\lim\limits_{x\to\infty} \sqrt{x}\cdot \sin\left(\dfrac{1}{x}\right)$.

3. Find $\lim\limits_{x\to 0} \dfrac{\sin(\sin(\sin x))}{x}$. Justify your answer.

4. Use the Intermediate Value Theorem to show that the following equations have a solution in a given interval I. Justify your answers.

(a) $x^5 - 4x^2 + e^x = 0,\ \ I = (-1, 1)$;

(b) $e^{\sin x} - \sin(\cos x) = 1,\ \ I = (0, \frac{3\pi}{2})$.

[Hint: Consider the restriction to a suitable subinterval of I.]

Bonus. Use the definition of derivative to prove that, if functions f and g are differentiable at a point a, then so are $f\cdot g$ and f/g (provided $g(a)\neq 0$), and their derivatives at a are given by the formulas:

$$(fg)'(a) = f'(a)g(a) + f(a)g'(a) \quad \text{and} \quad \left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{[g(a)]^2}.$$