Problem Set 1

due: May 19

1. Recommended practice problems from Sections 1.1–1.3.

2. Find the domain and range of f. Justify your answer.

(a) $f(x) = \ln(\arctan(e^x - 2015))$;

(b) $f(x) = \sin(\sqrt{\pi x - 4x^2})$;

(c) $f(x) = \ln\left(\frac{e^x + e^{-x}}{2}\right)$.

3. (a) Use the definition of injectivity to show that, if functions f and g are injective, then so is $g \circ f$.

(b) Use the definition of the inverse function to show that, if functions f and g are injective, then the inverse $(g \circ f)^{-1}$ is equal to the function $f^{-1} \circ g^{-1}$.

(c) Use part (b) to find the inverse of the function

$$h(x) = \frac{\ln x + 3}{5 - \ln x}\,.$$

Bonus. Let f be a one-to-one function. Prove that:

(a) If f is increasing, then so is f^{-1}.

(b) If f is decreasing, then so is f^{-1}.

(c) If f^{-1} is increasing, then so is f.

(d) If f^{-1} is decreasing, then so is f.

[Hint: Argue by contradiction.]