A NOTE ON THE THERMAL HISTORY OF THE EARTH AND THE POSSIBLE ORIGIN OF A SOLID INNER CORE

A. E. Beck

There have been a number of papers written on the thermal history of the earth (Jacobs 1961; Lubimova 1958; MacDonald 1959) which take into account the heat transfer by radiation and conduction; more recently Runcorn (1963) has planned to take into account the heat transfer by convection. The two principal sources of heat have been assumed to be gravitational potential energy and radioactive disintegration. Some of the theories have assumed an initially uniform, others an initially nonuniform, distribution of radioactive sources. However, almost all authors have assumed implicitly a uniform distribution of both the initial gravitational potential energy (GPE) available at the time of accretion and of the subsequent energy available from gravitational reorganization. Ringwood (1960) recognized that the accretion energy available was not uniform, but in effect he used a method of successive averaging by taking a series of masses and finding the average energy available throughout each mass. It is the purpose of this note to investigate the effect of taking into account the essentially nonuniform distribution of energy per gram available from both the initial energy of accretion and the subsequent energy released during gravitational reorganization.

The GPE of the present earth is taken to be -2.48×10^{39} ergs (Beck 1961) and represents the total gravitational energy available during the whole history of the earth. What proportion of this energy comes from the initial accretion process and how much from the subsequent gravitational reorganization depends on the assumptions made about the accretion process and the density of the original material.

For the purposes of this note, it is assumed, following Urey (1952), that the earth was formed by meteoritic aggregation from material which has a density σ at zero pressure. It is also assumed that the time taken for accretion is small (10^8 years), compared with the age of the earth (4.5×10^9 years), that the earth originally formed as a sphere of uniform density σ, and that the mass of the earth has not changed appreciably since its formation.

As pointed out by Beck (1962), it is possible to determine the total amount of energy available from gravitational reorganization by finding the difference between the GPE of the present earth and that of the original earth of uniform density σ. In fact the GPE of the original earth gives, in a sense, the energy available from gravitational reorganization from a sphere of zero density (finite mass, infinitely dispersed) to one of uniform density σ.

The important quantity required is not the total energy available for the whole earth, but rather the energy per gram available in the various regions of the earth. The accretion energy per gram, E_1, for the region $R_1 < r < R_2$ of a uniform sphere is given by:

$$E_1 = \frac{4}{5} G\pi\sigma \left[\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right], \tag{1}$$

where G is the gravitational constant.

The energy per gram available from gravitational reorganization, E_2, is then found from the difference between the energy given by equation (1) and the contribution to the GPE of the present earth of the same mass as is involved in equation (1), but which will now lie between different limits of r because of the reorganization of material.

Relevant values for two cases have been worked out and are presented in Table I. Two values of σ (2.85 g cm^{-3} and 4.4 g cm^{-3}) have been chosen to represent the limits to the range of possible values of σ. The contributions of GPE for each region of the present earth were calculated from the density distribution fitted by Beck (1960) to Bullen's (1950) data. A different present-earth model would not alter the figures significantly.

TABLE I
Energy available in different regions of the earth owing to accretion and gravitational reorganization
(All energies given in ergs/10^{39})

		If same mass is of uniform density σ g cm^{-3}			
		$\sigma = 2.85$		$\sigma = 4.4$	
Region, km	Contribution to present GPE	Contribution to original GPE	Difference available as energy	Contribution to original GPE	Difference available as energy
$0 < r < 1389$	−0.0084	−0.0078	0.0036	−0.0055	0.0029
$1389 < r < 3471$	−0.4256	−0.2662	0.1594	−0.3077	0.1179
$3471 < r < 4200$	−0.2795	−0.1890	0.0905	−0.2185	0.0610
$4200 < r < 5000$	−0.4688	−0.3363	0.1325	−0.3886	0.0802
$5000 < r < 6291$	−1.2189	−0.9425	0.2764	−1.0893	0.1296
$6291 < r < 6371$	−0.0782	−0.0625	0.0157	−0.0724	0.0058

If these data are now plotted in terms of ergs/g versus the half-mass radius (i.e., that radius which divides a shell into two equal masses) of each region, Fig. 1 is obtained. The horizontal line plotted on the same figure represents the energy per gram required to raise the temperature of the material to 3000° K and then to melt it. The melting point will in fact be a function of the pressure, and hence of the radius, but at radii of less than 3500 km the melting point of the undifferentiated material is likely to be considerably greater than 3000° K, so that this temperature is a conservative choice. A heat capacity of 1.3×10^7 ergs/g/° K (Uffen 1952) and, in the absence of any other data, a latent heat of fusion for iron of 5×10^9 ergs/g (Verhoogen 1961) have been used.

In view of the present preference of a time of origin of about 10^{10} years ago for nucleosynthesis, it is unlikely that the short-lived radioactive isotopes have contributed significant quantities of energy to the earth. It can also be shown from MacDonald's (1959) data that the energy contribution from

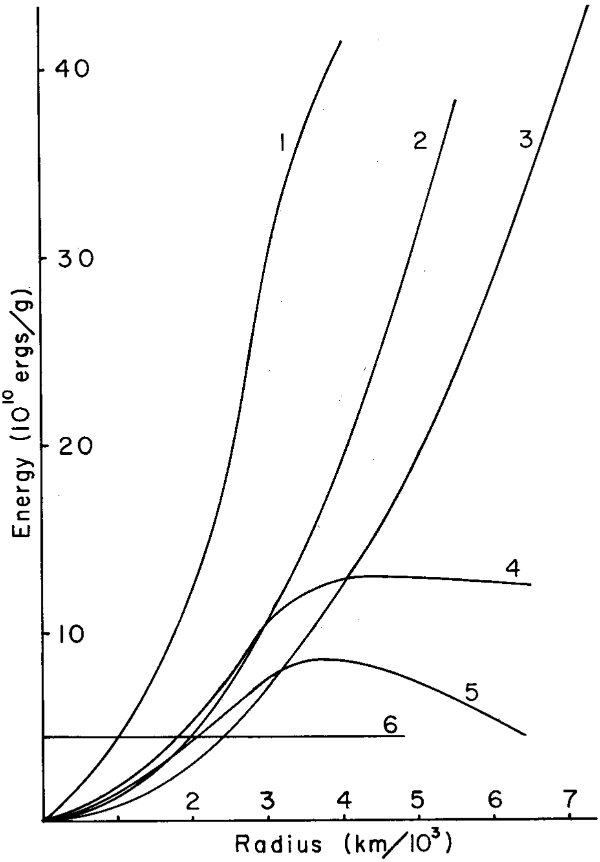

FIG. 1. Available gravitational energy per gram versus radius: 1, total energy of present earth; 2, accretion energy for σ = 4.4; 3, accretion energy for σ = 2.85; 4, energy from gravitational reorganization for σ = 2.85; 5, energy from gravitational reorganization for σ = 4.4; 6, energy required to raise material to 3000° K and then to melt it.

the long-lived isotopes is insignificant during the accretion process. Energy contributions from radioactive isotopes are therefore ignored for this stage.

Subsequently it can be seen that the maximum energy per gram, E_2, available from gravitational reorganization occurs at a radius of approximately 4500 km if $\sigma = 2.85$ and at about 3500 km if $\sigma = 4.4$. In both cases the available E_2 tends to decrease, although at different rates, with increasing radius in the mantle. The important point is the rapid increase of E_2 in both cases as r increases; the maximum value of E_2 occurs in the mantle and is between four and six times that available in the core. It thus appears that in those cold-origin theories which postulate that all or most of the accretion energy, E_1, is radiated into space, the mantle is far more likely to melt because of the energy available from gravitational reorganization than is the core.

Since all curves cross the fusion energy line in the vicinity of the inner core – outer core boundary, it is interesting to speculate on the possibility that the earth's inner core has been solid since its formation.

If the inner core did not reach its fusion temperature during the formation of the earth, its temperature will, of course, have since increased by inward conduction of heat through the zone, probably a few tens of kilometers thick, which is at the melting-point temperature immediately surrounding the solid core. If it is assumed that all the energy available from the core went into heating it (i.e., that none is lost by radiation, phase changes, etc. during the formation), then the curves shown will reflect, with an appropriate change in scale, the temperature gradient in the core. The argument is clearly unrealistic, but it strongly favors a melting of the core, and it is now instructive to estimate how long it would take to raise the temperature at $r = 500$ km to a temperature v of 2900° K.

Outside the core the material will be molten and it seems reasonable to suppose that convection currents will cause the temperature gradient to flatten off so that the core is effectively surrounded by a medium which is at a constant temperature $V = 3000$° K. Ignoring the energy that has to be transferred to supply the latent heat of fusion, an assumption which favors a more rapid increase of temperature with time than would otherwise be true, it is possible to avoid an elaborate treatment and use the curves given by Carslaw and Jaeger (1959, p. 234) for a sphere at zero initial temperature and with a constant surface temperature.

The temperature distribution indicated by any one of the curves of Fig. 1, curve 2 for example, may be regarded as the distribution that would be observed in the above case after a time t_i, where t_i is found from the parameter $\kappa t_i/a_c^2$, of the Carslaw and Jaeger curve which best fits curve 2; κ is the diffusivity and a_c denotes the core radius given by the intersection of curve 2 with line 6. If $r = 500$ km, $v = 2900$° K, and V is the temperature at the surface of the sphere (i.e., 3000° K at $r = a_c$), r/a_c and v/V will give a value of $\kappa t_0/a_c^2$, where t_0 is the time required to raise the temperature of the sphere from zero to the temperature distribution which gives $v = 2900$° K at $r = 500$ km.

The time, t_m, necessary to raise the temperature at $r = 500$ km to 2900° K may therefore be found from the equation

$$t_m = \frac{a_c^2(t_0 - t_i)}{\kappa}\,. \tag{2}$$

A value of $\kappa t_i/a_c^2 = 0.06$ or 0.07 seems to fit most of the curves.

The value of κ is uncertain. The surface value for most rocks is 0.01 cm^2 sec^{-1} and Uffen (1952) believes that it may increase by as much as a factor of four at the core–mantle boundary. The value assumed here is that $\kappa = 0.05$ cm^2 sec^{-1}. A value of a_c can be found for each of the cases from the intersection of line 6 with the appropriate curve so that t_m can be found in each case. For the worst possible case (curve 1), i.e., regarding the earth as having been formed instantaneously with its present-density distribution from a zero-density distribution, t_m is found to be 1.71×10^9 yr. For curves 2 and 4 the corresponding times are found to be 7.75×10^9 yr and 5.23×10^9 yr, respectively.

It might be argued that in applying the curves for a sphere of zero initial temperature to the present case, a radius larger than a_c should be taken to allow for the possibility of no convection in the molten portion and hence temperatures above the melting point gradient. A second calculation demonstrates the possible effect. When spheres of radius $(a_c + 1000)$ km are taken (implying temperatures of the order of 10,000° K at that radius), the values of t_m are 0.76×10^9 yr, 2.85×10^9 yr, and 2.85×10^9 yr, respectively, for the above three cases. If all the energy released by radioactive decay since the formation of the earth is allowed for by displacing line 6 downward by 2×10^{10} ergs/g (MacDonald 1959), the effect is roughly to halve the above values of t_m.

In all cases, if r were taken to be 100 km instead of 500 km, t_m would be increased significantly.

Thus it appears that it is very unlikely that the earth reached a completely molten state 4.5×10^9 years ago, as suggested by Ringwood (1960). Furthermore, in view of the unrealistic assumptions made which strongly favor melting of the core, it appears that the case for a completely molten core at any stage of the earth's history is borderline and it seems possible that not only has the earth always had a solid inner core, but that the central portion of it may contain compressed primary material.

In any event, it appears that gradual melting of the inner core takes a long time. If this continued after the mantle solidified from the bottom upwards, appreciable quantities of the radioactive material coming from the inner core as it differentiated may have been trapped in the outer core. It would then be possible for radioactive isotopes to be distributed throughout the inner and outer core, although the mantle may be substantially free of them.

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DEPARTMENT OF GEOPHYSICS,
UNIVERSITY OF WESTERN ONTARIO,
LONDON, ONTARIO.